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# The Specific Gravity of Sea Water

# and the

Ghyben-Herzberg Ratio at Honolulu

by

Chester K. Wentworth

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## Chester K. Wentworth



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## AUTHOR'S NOTE

**T**HIS STUDY was made by the Board of Water Supply as a part of a comprehensive and hydrologic investigation of the Honolulu watershed. Principles discussed are involved in the behavior of underground water bodies in many oceanic islands and coastal areas; the specific data presented are of particular practical importance to students of water supply in Hawaii. As a contribution both to widely applicable theory and to the growing body of measurements of practical local utility, it is especially fitting that it be published by the University of Hawaii.

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# The Specific Gravity of Sea Water and the Ghyben-Herzberg Ratio at Honolulu

The GENERAL principle of floatation of a fresh water lens, supported by underlying salt water, within the rocks of a permeable oceanic island or sea coast, was first set forth by Badon Ghyben in 1889.<sup>1</sup> It was independently recognized and described by Herzberg in 1901, both these writers having based their conclusions on observations made along the coast of Holland.<sup>2</sup> Such a condition of balance between fresh water and sea water is exemplified to a remarkable degree in the Hawaiian Islands and it is not surprising that after data became available from numerous artesian wells drilled after 1880, the same explanation was worked out independently by W. D. and A. C. Alexander, in Honolulu.<sup>3</sup> The first statement of this theory, now known as the Ghyben-Herzberg principle, known to have been made in a geologic publication in America was that by Brown, in connection with his study of coastal ground water.<sup>4</sup>

<sup>4</sup> J. S. Brown, "A Study of Coastal Ground Water," U. S. Geol. Survey, Water Supply Paper 537, pp. 16-17, 1925.

<sup>&</sup>lt;sup>1</sup> W. Badon Ghyben, "Nota in verband met de voorgenomen put boring nabij Amsterdam," K. Inst. Ing. Tijdschr., 1888-1889, p. 21, The Hague, 1889.

<sup>&</sup>lt;sup>2</sup> Baurat Herzberg, "Die Wasserversorgung einiger Nordseebader," Jour. Gasbeleuchtung Wasserversorg., Jahrg. 44, Munich, 1901.

<sup>&</sup>lt;sup>3</sup> That the principle was recognized by the Alexanders seems well established by facts presented by Stearns, though it is more implied than explicitly stated in an article by W. D. Alexander in the Pacific Commercial Advertiser for October 9, 1908. H. T. Stearns, "Geology and Ground-Water Resources of the Island of Oahu, Territory of Hawaii," Division of Hydrography, Bulletin 1, p. 256, 1935.

Further statements and discussion of the principle with reference to Hawaii have been presented, among others, by Palmer,<sup>1</sup> Mc-Combs,<sup>2</sup> Kunesh,<sup>3</sup> and Stearns.<sup>4</sup>

It is hardly necessary to restate the principle but it is desired to call attention to certain points that are often overlooked. One of these relates to the combination of static dynamic equilibrium which is exemplified in the Ghyben-Herzberg lens of a permeable island. When it is stated that the head of fresh water above sea level, (H), the depth below sea level to which fresh water extends, (D), and the specific gravity of salt water in relation to fresh water, (SG), are related as in the following formula:



a static equilibrium, as that in a U-tube, is implied. This equation is a complete statement only if there is no movement of the water, which is not true at Honolulu. The lens-shape of the body of fresh ground water, where it is non-artesian, involves a slope seaward of the water table. This slope causes a seaward motion of the fresh water which would eventually level off the whole top of the lens. and thus eventually drain off all the fresh water, including the subsea level part of the lens, if no more rain water were added to its top. Moreover, with heavy rainfall addition, and through the action of such slope movement, the basal water table at a given point may acquire a head somewhat in excess of the amount momentarily balanced by the greater density of sea water. In general it may be presumed, that whereas in times of great rainfall addition, there may be a marked surcharge of water-table head over the position of balance (what might be called a dynamic surcharge), the production, through the depletion of the stored water by slope movement, during times of reduced rainfall, of a corresponding degree of

<sup>&</sup>lt;sup>1</sup> H. S. Palmer, "The Geology of the Honolulu Artesian System," Supplement to Report of the Honolulu Sewer and Water Commission, p. 19, 1927.

<sup>&</sup>lt;sup>2</sup> J. McCombs, "Methods of Exploring and Repairing Leaky Artesian Wells on the Island of Oahu, Hawaii," U. S. Geol. Survey, Water Supply Paper 596, pp. 7-8, 1927.

<sup>&</sup>lt;sup>3</sup> Honolulu Sewer and Water Commission, Report to Legislature, January, 1929, Plate 10.

<sup>4</sup> Stearns, op. cit., pp. 237-238, 253-256, 1935.

dynamic deficiency at the basal water table is far less likely. This is due to the greater impediment offered to the movement of water in the saturated rock mass below the water table as compared to the freer percolating movement by which water may be added to the top of the water table. From this discussion it will be seen that the general form of the lens-shaped mass of fresh water above and below sea level indubitably does correspond to the static equilibrium fixed by the density relations of fresh and salt water. But static equilibrium does not prevail, because the slope of the water table causes a constant spreading movement in a seaward direction, and because of irregularity of rainfall addition and loss by the incessant lateral movement. Hence the water table can never have the perfect adjustment to the existing pressure of sea water which is implied by the static equation. In short, as in many other complex natural systems, dynamic and static characteristics are combined. However, in view of the approximate correctness of the static equation, it is important for practical purposes to know the average specific gravity of sea water referred to fresh water as accurately as possible.

The density of sea water in the ocean generally varies from 1.020 to 1.030. In the North Sea, where the first studies of Ghyben and of Herzberg were made, the value used for density was 1.027.<sup>1</sup> Only a few determinations of the specific gravity of sea water in the vicinity of the Hawaiian Islands appear to have been made. Several years ago a number of series of sea water samples were collected by the U.S. Coast and Geodetic Survey and turned over to the Scripps Institution for study.<sup>2</sup> Results of specific gravity determinations on these samples have recently been furnished the writer, and are in part quoted on another page of this paper.

Carl B. Andrews, in the first structural description of the artesian system, in 1909, gives for the ratio of fresh water below sea level to that above the value 38.43, which corresponds to a specific gravity of sea water of 1.02602.<sup>8</sup> He did not then state the source of his information on the specific gravity of sea water but recently

<sup>&</sup>lt;sup>1</sup> Brown, op. cit., p. 17, 1925.

<sup>&</sup>lt;sup>2</sup> L. O. Colbert, Director, U.S.C. & G.S., Letter dated May 13, 1938.

<sup>&</sup>lt;sup>3</sup> Carl B. Andrews, "The Structure of the Southeastern Portion of the Island of Oahu, Hawaiian Islands," (Rose Polytechnic Institute, Master's Thesis, 1909. Copy in Water Commission Report, Public Archives, 1909.)

informed the writer that at that time he had had occasion to make a number of determinations in a specific gravity bottle or picnometer in connection with a scheme for recovery of salt from sea water.<sup>1</sup> It therefore appears that he was in possession of fairly exact data.

Palmer, in 1927, states that "the sea water around the Hawaiian Islands has a specific gravity of about 1.025."<sup>2</sup> This value corresponds to a Ghyben-Herzberg ratio of 1:40, which has been a commonly accepted "round number" since that time. In 1927, McCombs reported a determination of the specific gravity of sea water as 1.024, furnished by H. L. Lyon.<sup>3</sup>

This value was accepted by Stearns, who adds the report of two samples, one from Pearl and Hermes Reef, and one from outside of Pearl Harbor, Oahu, with specific gravities of 1.0222 and 1.0220 at 24 degrees Centigrade (75.2° F). These specific gravities are corrected by Stearns to 1.0239 at 60° F (15.56° C) and 1.0246 at 50° F (10°C).<sup>4</sup> From this procedure he concludes that "it is evident that 1.024 is the practical figure to use for the water off Oahu." There are several errors of reasoning here. First, it is not clear why Stearns corrects to temperatures of 60 and 50 degrees Fahrenheit, since the mean temperature of ground water at 500 feet below sea level certainly does not vary a degree from 72 degrees, and mean ground water temperatures even so low as 65 degrees are not encountered at elevations below 1,000 feet. Second, the only significant specific gravity in such work is the ratio between weights of equal volumes of salt and fresh water respectively at the same temperature or at respective natural temperatures. If the value given by Stearns, quoted from Collins, was determined with reference to fresh water at the same temperature, then the correction applied should be the differential correction which takes account of the fact that salt water contracts more rapidly than does fresh water as the temperature falls (Figure 1). This differential correction is about 0.000046 per degree Centigrade. It appears that Stearns used the four-times-as-great correction which would hold for distilled water. Or, if, on the contrary, the specific gravity quoted from Collins is

<sup>&</sup>lt;sup>1</sup> Personal communication, June 24, 1938.

<sup>&</sup>lt;sup>2</sup> Palmer, op. cit., p. 19, 1927.

<sup>&</sup>lt;sup>s</sup> McCombs, op. cit., pp. 7-8, 1927.

<sup>4</sup> Stearns, op. cit., Footnote, p. 255, 1935.



Diagram showing thermal expansion of distilled water and sea water. The inset at lower right fits at left edge of main graph. For convenient comparison the curves are drawn to show volume change in relation to assumed par volume at 0° Centigrade. The double line for sea water represents extremes of salinity well outside the fluctuation found in surface water at Honolulu. For practical purposes between 20° and 30° the two sea water curves may be regarded as of identical slope and curvature. Distilled water contracts to a minimum volume at 4° and thence expands at higher temperatures in a manner similar to the salt water. But the curves are not of precisely the same slope at the same temperatures, the differential greater expansion of sea water in this range of salinity being shown by the curve marked differential rate. (Data from Knudsen, Hydrographical Tables, and Physics and Chemical Handbook.)

referred to fresh water at 4° Centigrade (as is customary in much physical work) there would be required a still more complicated correction leading to a correct value much higher than that adopted by Stearns. From the actual figures found, there is reason to suspect that the Collins' determination was referred to fresh water at 4° Centigrade. From these various facts it appears that the agreement at 1.024 found by Stearns between the two samples determined by Collins and that reported by Lyon is entirely fortuitous and the value itself is open to considerable suspicion. The same reasoning applies to the Ghyben-Herzberg ratio of 1:42 derived from the specific gravity value and used throughout the Oahu report.

Apparently following Stearns, Sohlberg has used the value 1.024 for specific gravity and derived the strict value of 1:41 2/3 for the Ghyben-Herzberg ratio, a refinement of computing only, based unfortunately on a figure already suspect.<sup>1</sup> In the meantime, this office has continued to use the convenient round number of 1:40, based on the value given by Palmer in 1927. Relations between specific gravities and Ghyben-Herzberg ratios are shown graphically in Figure 2.

In the belief that the practical importance of this ratio justified



Graph showing relation between specific gravity ratio and the Ghyben-Herzberg ratio. The ordinate scale is the logarithm of the Ghyben-Herzberg ratio, and the abscissa scale is the logarithm of the specific gravity minus one, hence the locus becomes a straight line.

<sup>1</sup> R. G. Sohlberg, "The Hawaiian Ground-Water Province," (section in C. F. Tolman, *Ground Water*, McGraw-Hill, 1937) p. 543.

more extended, first-hand local measurements of the specific gravity of sea water, the present writer commenced in March, 1938, collecting and determining such samples. Many of the samples were collected in connection with his non-official studies of marine benches on Oahu shores, but subsequently this was approved as an official project and a few additional samples collected and determined. Twenty-six samples were collected by ships of the Inter-Island Steam Navigation Company at various points in Hawaiian waters. Thanks are due to Herbert Martin, Port Captain of the Inter-Island Company, who kindly made arrangements for collection of these samples, and to the masters of the several ships for collecting them.

Most of the samples amounted to a quart each, permitting analysis in case unusual features were encountered, and a few of the samples have been analyzed for chloride by L. T. Bryson, as shown in the following table.

Lab	Field1	<u> </u>			
No.	Number	Chloride <sup>2</sup> Milligrams/liter	Lab. No.	Field <sup>1</sup> Number	Chloride <sup>2</sup> Milligrams/liter
11013	632	19,460	11054	685	19,660
11014	636	19,670	11055	687	19,580
11015	648	19,720	11221	736	19,700
11016	652	19,440	11222	738	19,730
11017	654	19,590	11223	748	19,760
11018	656	19,540	11224	749	19,760
11019	65 <b>7</b>	19,400	11252	762	19,580 <sup>s</sup>
11020	661	19,650	11253	763	19,610 <sup>3</sup>

Complete mineral analyses of two samples (Nos. 762 and 763) have been made and are shown below.

#### MINERAL ANALYSES OF SEA WATER <sup>4</sup>

#### Physical data:

3

Laboratory No.	11252 5	11253 °
Field No.	762	763
pH value	7.9	8.2

Analyses by L. T. Bryson.
1 For localities and other data, refer to these numbers in the table below.
2 Chloride with bromide and iodide.
3 Complete mineral analyses are reported below.
4 Analyses by L. T. Bryson.
5 Collected from middle of Hawaii Channel, 21 miles from each light (Hanamanioa and Kauhola) by B. A. Pfeiffer, 2nd mate. Steamer Waialeale.
6 Collected from position 30 miles west of Kaena Point, by E. L. McManus, master, Steamer Waialeale.

Specific gravity:		
at 30°/4° C	1.02230	
at 20°/20°C	1.02651	

## Analyses, in parts per million:

Silica (SiO <sub>2</sub> )		3
Iron oxide (Fe <sub>2</sub> O <sub>8</sub> )		10
Alumina (Al <sub>2</sub> O <sub>8</sub> )	1.2	1.1
Calcium (Ca)	405	405
Magnesium (Mg)	1,305	1,315
Sodium (Na)		10,540
Potassium (K)		400
Free carbon dioxide (CO <sub>2</sub> )	4	1
Bicarbonate (HCO <sub>a</sub> )	135	138
Sulphate (SO4)		<b>2,6</b> 10
Chloride <sup>†</sup> (Cl)		19,150
Bromide (Br)		52
Phosphate (PO <sub>4</sub> )		3
Total solids		34,530
Alkalinity (as CaCO <sub>s</sub> )	110	115
Total hardness (as CaCO <sub>s</sub> )	6,360	6,400

## Balance of bases and acids, in milliequivalents:

Calcium	20.2	
Magnesium	107.3	
Sodium	461.8	
Potassium	8.7	10.2
Error	1.5	
Total	 596.5	
Bicarbonate	2.2	2.3
Sulphate	54.5	54.3
Chloride	539.2	
Bromide	.6	
Phosphate	.0	0
Total	596.5	

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## Hypothetical combinations, in grains per U. S. gallon:

Silica	.41	.17
Iron oxide	.01	.01
Alumina	.07	.06

7 Includes iodide.

Calcium carbonate	6.42	6.71
Calcium sulphate	71.43	71.04
Calcium phosphate	trace	trace
Magnesium carbonate		
Magnesium sulphate	128.12	127.77
Magnesium chloride	196.56	199.06
Sodium carbonate	<b></b>	<b></b>
Sodium sulphate		•
Sodium chloride1	1,567.191	,562.09
Potassium chloride	35.23	41.32
Potassium bromide	4.16	4.86
-		
Total dissolved solids	2,009.60	2,013.09
Incrusting solids, lbs. per 1,000		
U. S. gallons	11.19	11.14
Non-incrusting solids, lbs. per		
1,000 U. S. gallons	275,90	276.44

Determination of specific gravity was by 100 C.C. picnometer on a balance reading to 0.0001 gram by chain vernier and after samples had stood at least 24 hours in the laboratory. Before each series of sea water weighings, and after most of the series, a weighing of distilled water was made, so that at the laboratory temperature, which was recorded before and after the series, the picnometer was directly calibrated. In this way the specific gravities as derived were referred to distilled water at the same temperature, ranging from 26° to 30° C. Correction of these values to the adopted value of 22° C. took account within this range of the contraction of distilled water by about .00027 per degree Centigrade, and of sea water by about .00031 per degree Centigrade, which amounts to a net increase of specific gravity of approximately 0.000046 per degree lowering of temperature.

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Several series of check readings in pairs or larger groups showed a probable error of the laboratory determination of about 0.000016 and samples taken at the same place and time in the field show a probable error of sampling of about 0.000025.

Num		Specific	Growity	Num		Secultic	Crowiter
ber	<b>Place</b> <sup>1</sup>	T/T <sup>2</sup>	22/22 <sup>3</sup>	ber	Place <sup>1</sup>	T/T <sup>2</sup>	22/22 <sup>3</sup>
Collect	ted 3/28/	/38. 1ab. tem	p. 28.8	676	<u>т</u>	1.02373	*
632	Α	1.02621	1.02652	Collec	ted 4/10/	38. lab. tem	D. 268
633	A	1.02623	1.02654	677	T	1.02370	*
634	В	1.02634	1.02665	678	Ū	1.02380	*
635	B	1.02631	1.02662	679	Ŭ	1.02345	*
636	č	1 02646	1.02677	680	v	1.02379	*
637	č	1.02639	1.02670				
638	õ	1.02645	1.02676	Collec	ted 4/10/	38, lab. temp	p. 27.2
630	ñ	1 02636	1.02667	681	V	1.02368	*
C.11.		2.010000	20.5	682	W	1.02501	*
Conect	ted 4/3/3	1 0 25 96	. 29.3	683	W	1.02503	*
041	E	1.02580	1.02021	Collec	ted 4/4/3	88, lab. temp	. 26.8
042	E	1.02588	1.02020	684	х	1.02608	1.02630
643	E	1.02601	1.02030	685	Y	1.02615	1.02637
645	F	1.02600	1.02035	686	Z	1.02603	1.02625
646	F	1.02595	1.02030	Collec	ted 4/5/3	8. lab. temn	. 26.8
647	F	1.02615	1.02050	687	AA	1.02586	1.02608
648	G	1.02620	1.02055	Collee	tod 1/11/	28 101 40m	20.2
649	G	1.02619	1.02654	600	100 4/14/ 1919	36, Iaų. temp	1.02656
650	Н	1.02610	1.02645	600		1.02022	1.02056
651	Н	1.02614	1.02649	600		1.02019	1.02053
652	1	1.02579	1.02614	601	DD FF	1.02008	1.02642
653	I	1.02583	1.02618	091	EE	1.02619	1.02053
Collect	ted 3/30/	38, lab. temp	. 29.5	Collec	ted 4/15/	'38, lab. tem <u>r</u>	p. 29.3
654	J	1.02590	1.02625	692	FF	1.02387	*
Collect	ted 3/31/	38. lāb. temu	29.5	693	FF	1.02387	*
655	K	1 02584	1.02619	694	GG	1.02385	+
000			20.5	695	GG	1.02364	*
Collect	ted 4/3/8	88, Jab. temp	. 29.5	Collec	ted 4/15/	'38. lab. tem	D. 30.0
050	L	1.02579	1.02014	696	нн	1.02558	1.02595
657	м	1.02553	1.02588	697	п	1.02559	1.02596
Collect	ted 3/31/	38, 1ab. temp	. 29.5	698	II	1.02568	1.02605
658	N	1.02583	1.02618	699	Î	1.02570	1.02607
Collect	ted 4/1/3	8. lab. temp	. 29.5	Collec	ted 4/16/	38 Jab tem	30.0
659	0	1.02583	1.02618	700	НН	1 02573	1 02610
660	P	1.02507	*	701	11	1 02566	1.02010
661	0	1.02587	1.02622	Caller		1.02500	1.02003
0.11	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~		<b>A</b> ( )	702	The	36, Iab. tem	p. 30.0
Collect	ted 4/10/	Jo, lab. temp	. 20.0		<b>D</b>	1.02390	1.02627
0/U 671	Б	1.02303		Collec	$\frac{4}{17}$	38, lab. tem	p. 30.0
0/1 674	ĸ	1.02540	-	704	C	1.02588	1.02625
0/4	2	1.02457	-	705	C	1.02578	1.02615
675	S	1.02459	*	706	В	1.02590	1.02627

## Specific Gravity Determinations on Sea Water Samples, Hawaii

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# SPECIFIC GRAVITY DETERMINATIONS ON SEA WATER SAMPLES, HAWAII (cont.)

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Num-	Placel	Specific	Gravity	Num-	Diacel	Specific	Gravity
707	- <u>1 late-</u>	1.02570	1.02(1)	720	1 Iacc-	1.02501	1.00(17
707	Б	1.02579	1.02010	739		1.02591	1.02017
708	A	1.02580	1.02617	740	JJ	1.02620	1.02646
709	A	1.02578	1.02615	Collec	ted 4/28/	'38, 1ab. tem	p. 27.7
Collect	ted 4/18/	38, 1ab. tem	o. 30.0	741	KK	1.02604	1.02630
710	JJ	1.02579	1.02616	742	LL	1.02612	1.02638
711	HH	1.02565	1.02602	743	MM	1.02611	1.02637
712	II	1.02558	1.02595	Collec	ted 5/1/3	8. lab. tem	. 27.7
Collect	ted 4/19/	38, lab. temp	. 28.2	744	NN	1.02619	1.02645
713	$\mathbf{H}\mathbf{H}$	1.02588	1.02617	Collec	ted 5/2/3	8. lab. temp	27.7
714	II	1.02581	1.02610	745	00	1 02607	1 02633
715	JJ	1.02596	1.02625	746	PP	1.02614	1.02000
Collect	ted 4/21/	38 Joh teme	28.2	747	00	1.02612	1.02638
716	HH	1 02583	1.02612	Colleg	+01 5/2/2	9 lab toma	777
717	IT	1 02554	1 02583	7/0		0, 1ab. temp	1.02652
718	TT	1 02590	1 02619	740	CC KK	1.02027	1.02033
Caller		20 1-1 (	20.0	747		1.02020	1.02034
Collect	tea 4/24/	Jo, Iab, temp	1.02620	Collec	ted 5/6/3	8, lab. temp	. 27.7
/19	D	1.02599	1.02028	750	TT	1.02613	1.02639
720		1.02602	1.02631	751	00	1.02618	1.02644
721	В	1.02599	1.02028	752	٧V	1.02610	1.02636
722	A	1.02003	1.02032	Collec	ted 5/14/	38, 1ab. tem	p. 26.7
720	n D	1.02002	1.02031	753	D	1.02600	1.02622
724	р С	1.02009	1.02030	754	D	1.02607	1.02629
763		1.02000	1.02029	755	D	1.02607	1.02629
120	D	1.02599	1.02020	756	D	1.02609	1.02631
Collec	ted 5/1/3	38, lab. temp	. 28.6	757	D	1.02625	1.02647
727	A	1.02617	1.02647	Collec	ted 5/15/	'38. 1ab. terr	n. 26.0
728	В	1.02650	1.02680	758	A	1.02616	1.02634
729	С	1.02618	1.02648	759	B	1.02626	1.02644
730	D	1.02615	1.02645	760	ĉ	1.02628	1.02646
731	BB	1.02612	1.02642	761	Ď	1.02616	1.02634
732	CC	1.02609	1.02639			/20 1 1	
733	DD	1.02611	1.02641	Collec	tea 5/11/	38, lab. tem	ip. 26.0
734	EE	1.02610	1.02640	/02	ww	1.02643	1.02661
Collec	ted 5/7/3	38, 1ab. temp	. 27.7	Collec	ted 5/13/	38, lab. tem	p. 26.0
735	Α	1.02629	1.02655	763	XX	1.02638	1.02656
736	в	1.02630	1.02656	Collec	ted 5/15/	38, lab. tem	p. 26.0
737	С	1.02629	1.02655	764	YY	1.02607	1.02625
738	D	1.02632	1.02658	765	ZZ	1.02617	1.02635

Average 104 determinations 1.026334 4

[ 13 ]

#### FOOTNOTES TO PRECEDING TABLE

<sup>1</sup> See table of localities which follows.

<sup>2</sup> Referred to distilled water at same temperature.

<sup>8</sup> Corrected for thermal expansion of both distilled water and sea water, net amount equals increase of 0.000046 in specific gravity per 1 degree Centigrade decrease in temperature.

<sup>4</sup> The standard deviation of individual measurements, regarding each as a random measurement of the value sought, is 0.000184 and hence the probable error .6745 x 0.000184 or 0.000124. Considered as a continuous series of measurements, the probable error of the mean is thus approximately 0.000012. As another approach to estimate validity of the average, the averages of two series, even and odd alternate members, were taken, giving 1.026326 and 1.026342, a deviation of each from the mean of 0.000016. This roughly confirms the estimate that the average of the above table, namely 1.026334, is not in error by more than 0.00002, as a determination of the sea water at the times and places listed, in relation to distilled water at 22° C. Referred to artesian water, which has a specific gravity averaging 1.00023, the apparent best value is 1.02610.

\* Following heavy rains; diluted by land water, not included in average.

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#### TABLE OF LOCALITIES

(referred to in preceding tables)

- A-Kuapo, off rocks.
- B-Wawamalu Ranch, off rocks.
- C-Koko Blowhole, off rocks.
- D-West of Koko Blowhole, off rocks.
- E-Waianae Coast, Kaneana Cave, off rocks.
- F-Keaau Beach, Waianae Coast.
- G-Waianae Peninsula, north side, off rocks.
- H-Waianae Peninsula, south side, off rocks.
- I-Coast below Puu o Hulu Kai, beach.
- J-9 miles E.S.E. of Diamond Head Inter-Island Steam Nav. Co.).
- K-5 miles E. by N. Kamalo Bay (I.I.S.N. Co.).
- L-2 miles N. by E. of Kohala Light (I.I.S.N. Co.).
- M-1 mile east of Pepeekeo Light (I.I.S.N. Co.).
- N-Kaumalapau Harbor, Lanai (I.I. S.N. Co.).
- O-Mahukona, Hawaii (I.I.S.N. Co.).
- P-Kailua, Hawaii (I.I.S.N. Co.).
- Q-1<sup>1</sup>/<sub>2</sub> miles southeast of Makahuena Light, Kauai (I.I.S.N. Co.).
- R-East of Waimea Bay, off rocks.
- S-West of Waimea Bay, off rocks.
- T-West of Waimea Bay, off rocks.
- U-West of Waimea Bay, off rocks.
- V-Rock Point west of Haleiwa, off rocks.
- W-Mokuleia Beach, beach.
- X-1 mile S.W. of Kaunakakai (I.I. S.N. Co.).
- Y-2 miles S.W. of Barber's Point (I.I.S.N. Co.).
- Z-10 miles south of Kaena Point (I.I.S.N. Co.).

- AA-5 miles S.E. of Kawai Point, Kauai (I.I.S.N. Co.).
- BB-First inlet, east side, Hanauma Bay.
- CC-Second inlet, east side, Hanauma Bay.
- DD-Third inlet, east side, Hanauma Bay.
- EE-Bench pool, seaward from natural arch, east side, Hanauma Bay.
- FF-Kaneohe Bay, east side.
- GG-Kaneohe Bay, east side.
- HH-Waikiki sea wall, center.
- II-Waikiki sea wall, east end.
- JJ-Black Point, foot of steps.
- KK—1 mile north of Kalolii Channel Buoy (I.I.S.N. Co.). (I.I.S.N. Co.).
- LL-5.5 miles south of Kaunakakai
- MM-2 miles south of Laau Point, Molokai (I.I.S.N. Co.).
- NN-2 miles east southeast of Kinau (I.I.S.N. Co.).
- OO-11 miles south by east from Makapuu Light (I.I.S.N. Co.).
- PP-2 miles south by east from Laau Point (I.I.S.N. Co.).
- QQ-9.5 miles south of Kaena Point (I.I.S.N. Co.).
- RR-6 miles east of Ninini Point (Nawiliwili) (I.I.S.N. Co.).
- SS-12 miles west of Kaena Point (I.I.S.N. Co.).
- TT-5 miles south by west from Kaunakakai (I.I.S.N. Co.).
- UU-8 miles west by north from Lahaina Light I.I.S.N. Co.).
- VV-8 miles northwest from Molokini Light (I.I.S.N. Co.).
- WW-Middle of Hawaii Channel, 21 miles to each light (I.I.S.N. Co.).

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#### TABLE OF LOCALITIES (continued)

XX-30 miles west of Kaena Point	ZZ-12 miles east southeast of Ma-
(I.I.S.N. Co.).	kapuu Head (Collected by
YY-7 miles southeast of Makapuu	C. D. Tarleton).
Head (Collected by C. D.	
Tarleton).	

Certain conclusions can be drawn by regrouping the data. In the following table, most of the determinations on shore samples are grouped by times, and by places.

Samples at nearly same time from different places			Samples from same place at different times			
Time	Number in Group	Spec. Grav. Average	Place	Number in Group	Spec. Grav. Average	
3/28	8	1.02665	A	9	1.02638	
4/3	12	1.02636	В	9	1.02645	
4/16	9	1.02617	С	8	1.02643	
4/24	7	1.02631	D	12	1.02639	
5/1	8	1.02648	F	3	1.02638	
5/7	6	1.02648	. нн	5	1.02607	
5/14	5	1.02632	II	Ê	1.02507	
5/15	4	1.02640	JJ	6	1.02620	
Average		1.02640		Average omitting HH, 11, and JJ	1.02641	
				Average with HH, II, and JJ	1.02628	

VARIATIONS OF GROUPS

In the third column, the differences in the several averages presented are time variations which have a range of 0.00048; in the sixth column are averages, differences among which are place variations. It is quite evident that sea water at stations HH, III and JJ, along the Waikiki-Black Point shore, is persistently lighter and of lower salinity than those of most other stations. Including these stations the place variations are about the same as the abovementioned time variation, but omitting these aberrant stations, the place variations have the low range of only 0.00007. As another approach the variation within the groups may be considered. In groups of measurements on samples from different places on the same day (within two or three hours) consisting of 4 to 12 measurements in a group, and a total of 8 groups and 59 measurements, the average root-mean-square deviation of specific gravities from the appropriate means of groups was 0.00010, maximum for any group 0.00014, minimum for any group 0.00004. Taken in the same way, groups of measurements on samples taken at the same place but at different times, total of 57 measurements, 3 to 12 in a group, 8 groups, the average root-mean-square deviation of measurements from appropriate means was 0.00018, maximum for any group 0.00033, minimum for any group 0.00007. Thus it is clear that time variations are practically twice the place variations, a representative figure for the former being 0.00018, for the latter 0.00010.

Considered by both methods it is evident that, barring exceptional conditions, there is more variation in the specific gravity of sea water from time to time, than there is from place to place, in the vicinity of Hawaii. This is believed to be due to the continually changing conditions of wind, currents, and tides, and to the consequent mixing and mingling of the layers of sea water of different compositions which are known to occur everywhere in the ocean at different depths. Hence a more valid average would be determined by taking samples over a long period of time at a few carefully chosen places, than at many places at one or a few times.

Because the density and salinity of the turbulent water near shore undoubtedly varies with the character of current and tidal movement and existing oceanic weather, it was thought that there might be systematic seasonal variations in mean salinity and density. Therefore a few additional samples have been taken from stations A, B, C, and D, during the months of October, November and December for comparison with samples taken from the same points during April and May. The number of these was insufficient to give a precise determination of any such seasonal variation but the determinations made on the few taken indicate that variation of the mean between these seasons was probably less than 0.00005. Only a long series of measurement would determine with certainty whether this is a characteristic seasonal difference, but in any event the difference is small from any practical standpoint. The above discussion applies more particularly to shore stations. The average of 77 shore stations compared at 22° and referred to distilled water is 1.026334, and of 27 samples collected by Inter-Island ships and other boats is 1.026333, a negligible difference.

Subsequent to making the larger part of the measurements reported here, a summary of determinations made at Scripps Institution on samples collected by the U.S. Coast and Geodetic Survey, was received.<sup>1</sup> These determinations consist of several series taken at depths from the surface downward to from 3,000 to over 5,000 meters, with corresponding temperature and salinity measurements. The specific gravities of the several surface samples, as collected and as corrected to 22 degrees and in relation to artesian water, are shown in the following table.

	Longitude	Date	Temp.	Specific Gravity	
Latitude				T°/4°	22°/22° Artesian Water
23-54 N	159-10 W	8/27/34	26.53	1.02310	1.02633
22-16½ N	158-34 W	8/27/34	27.21	1.02289	1.02642
22-53 N	151-15 W	9/20/29	26.14	1.02290	1.02621
21-47 N	155-31 W	9/22/29	26.72	1.022696	1.02607
23-26 N	159-27 W	10/3/29	26.91	1.022801	1.02624
23-52½ N	151-18 W	11/29/36	24.25	1.023781	1.02642
21-56 N	155-31 W	11/29, 30/36	24.53	1.023584	1.02631
				Mean	1.02628

SPECIFIC GRAVITIES OF SEA WATER

Each of the series reported includes samples from more than ten different depths and all show similar relationships of salinity, temperature and density to depth. All show a maximum salinity at the surface, which is reduced with depth to a minimum at some zone between 300 and 400 meters, and then increases moderately at greater depths. Temperature falls slightly in a zone near the surface then quite rapidly in the vicinity of 100 fathoms, and then at a decreasing rate to the value of about 4 degrees Centigrade at 1,000 fathoms and continues to fall slowly below that depth (see Figure 3). The densities at different depths depend much more on the temperatures than on the salinity. The mean given in the above table is .00018 greater than that found for both shore and ship sta-

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<sup>&</sup>lt;sup>1</sup> G. F. McEwen, letter dated July 21, 1938, with accompanying tabulation.



#### FIGURE 3

A composite graph showing approximate prevailing temperatures of sea water at various depths for the year (scale at top), relative densities of sea water at various depths as determined by the temperatures (scale at bottom), and cumulative relative densities of sea water from the surface downward to various depths (scale at bottom), and including effects of compression.

tions near the islands of Hawaii. It probably represents a less disturbed and more saline surface layer of the ocean in this latitude; the value found nearer shore is probably the product of more mixing, but is probably more representative of sea water involved in the Ghyben-Herzberg balance.

It is pertinent to consider the dilution effect of the daily increment of rainfall, greater on land than on the open ocean. If the ocean water, with surface currents of 2 miles per hour, be assumed to be changed once daily to a depth of 100 feet, the daily rainfall increment of 1/5 inch, or 1/60 foot, (Island of Oahu) amounts to only one part in 6,000. It would hence reduce the specific gravity of the sea water by less than 1 unit in the fifth decimal place, an effect far less than appears to result from mixing and churning of sea water from different layers. However important the dilution may be

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near rivers and submarine springs, it is concluded that its general effect is negligible.

The temperature for balancing salt and fresh water has been taken at 22° Centigrade (71.6° F.). Temperature measurements, so far made, indicate that the mean temperature of ground water in the range from sea level to 1,200 feet below sea level in the Honolulu artesian area is within  $\frac{1}{2}$  degree of this value. Temperatures of artesian water and basal water as measured to date are shown in the following table.

Station	Elevation	Degrees, Centigrade <sup>1</sup>		
Station		June	September	December
Waialae Shaft	10	21.60	21.79	21.68
Kaimuki Artesian	275	21.41	21.42	21.35
Beretania Artesian	600	22.01	21.99	22.01
Kalihi Artesian	425	20.95	20.95	20.93
Kalihi Shaft	28	21.49	21.60	21.52

Several assumptions may be made in regard to the temperature of the effective column of ocean water which balances the fresh water. In non-artesian areas where the zone of transition rises to sea level at the coast line and where the rock floor passes below sea level at a low angle, the salt water balancing the fresh water of the basal water lens is thought to be largely in the rock and to have the temperature of the artesian water, especially since the movement of water and hence of temperatures is from land water to sea water. But in a well-established artesian area, where the zone of transition may reach the ocean floor at considerable depth, it may be that part or nearly all of the balancing column of salt water is part of the free ocean and has its prevailing temperatures. Dr. H. S. Palmer has pointed out to the writer that the movement of ocean water around and past the islands at the surface and at considerable depths, is probably much more rapid than any movement of ground water outward to the submarine "skin" of the islands; hence it would appear that sea water temperatures are dominated by the currents and prevailing temperature profile of the ocean water.

Because of the unknown conditions seaward from the zone of transition at the point lying below any given well the problem of

<sup>&</sup>lt;sup>1</sup> C. K. Wentworth, Board of Water Supply, Seventh Biennial Report, p. 155, 1939.

the temperature and specific gravity of the salt water which is maintaining balance with the column of fresh water in the well can only be dealt with on a speculative basis. The chief alternatives are brought out in Figure 4. In case I it is assumed that along any given



#### FIGURE 4

Diagrammatic sketch illustrating two alternative concepts as to nature of conditions of balance between sea water and fresh water. These diagrams apply to a locality of slight artesian head, with comparatively thin cap rock, for more effective portrayal of the two alternatives.

line, as AB, the pressures of salt water in the rock are equal, that the column of water in the well can be considered as balanced against the column BC, salt water of the composition and at the temperatures which exist in the free ocean in the zone shown. This assumption would indubitably be correct in a strictly static system.

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In Case II it is assumed that the pressure exerted at A from the

salt water side is the result of summation of pressures exerted through a great many avenues extending from A to the surface of the sea (just as pressure at a given point in a system of pipes, in which water is moving, is a composite product of all the surrounding pressures and velocities). Two possible alternative routes of pressure transmission, AB'C' and AB"C", are indicated. In Case II it is also assumed that pressures in the salt water are possibly not equal along the line AB, in which case there will be resulting movement, and that in any event the pressures resulting from lines AB'C' and AB"C", because of the temperature peculiarities will not be exactly equal to that acquired along line ABC. Under these assumptions, the resulting mean pressure applied at A will be a composite result of combining in unknown proportions columns of water with the temperature of sea water, and with the temperature of the artesian bore, and with various intermediate combinations. A categorical choice of assumption does not seem justified.

Compression is another factor to be considered. Compression of fresh water, in the range between 0 and 1,200 feet, and between 20 and 25 degrees Centigrade, takes place at a rate approximately 148 x 10<sup>-6</sup> per 100 feet of overlying water. Compression of sea water in the same depth range and between 10 and 25 degrees Centigrade is at the rate of  $132 \times 10^{-6}$ , the differential rate being 16 x  $10^{-6.1}$ Since the fresh water is slightly more compressible it takes on at greater depths a relatively higher density and slightly reduces the specific gravity of sea water referred to it. Decrease in specific gravity of sea water at 1,200 feet by compression is 0.00019; cumulative decrease in the whole column to 1,200 feet is 0.00009. The latter figure is proportional to the square of the depth and is of negligible significance at depths under 500 feet.

The accompanying table shows the effect of combining columns at ground temperatures and those at free ocean temperatures in varying combinations and including the compression differential just discussed. These are further shown in Figure 5. (See also Figure 3.)

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<sup>&</sup>lt;sup>1</sup>Data for fresh water from *Physics and Chemistry Handbook*, p. 1053, 18th edition. Data for sea water from *International Critical Tables*, Volume 3, p. 439, 1928. Relationship between pressure and volume change in both cases deviates slightly from rectilinear, but the values will not be in error by more than 1 or 2 per cent through the specified range, the best slope at the middepth having been deduced by use of second and third differences from the values given in the tables.



VALUES OF SPECIFIC GRAVITY AND GHYBEN-HERZBERG RATIOS UNDER VARIOUS ASSUMPTIONS

Graph showing variations of Ghyben-Herzberg ratios due to changing effective specific gravity of salt water columns under the three different assumptions outlined in the accompanying table. (I) Assuming that balance is maintained by free ocean water. (II) Assuming that upper half of balancing column is free ocean water, and lower half is at temperature of ground water. (III) Assuming that balancing salt water column is in rock, with ground temperatures same as fresh water column. Effects of compression are also included.

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In view of these variations, it would perhaps be presumptuous to lay great stress on any particular standard value, but it appears that the average of values for a free ocean temperature balance, and that for ground temperature balance through the range of 0 to 1,200 feet are practically identical. In summary it is shown that for very shallow zones of balance, a free ocean balance gives a Ghyben-Herzberg ratio of 39.7 (nearly equal to the long used round number of 40), but for greater depths the ratio decreases almost proportionally to depth to reach the value of about 36.8 at a depth of balance of 1,200 feet. On the other hand assuming ground water temperatures throughout, there result practically constant Ghyben-Herzberg ratios at all depths. By the intermediate assumption of the third column of the table, the values range from 39.0 to 38.5. For those who wish a single, round number, without attempting to choose between these assumptions, and a value that will favor the greater depths so as to give a minimum error in feet at any depth, the value 38 seems to be the most representative.

Despite the impression the reader may gain, that in this paper many questions have been raised and positive answers offered for only a part, several definite results have been attained. First, the fundamental mean specific gravity of sea water near Hawaii referred at 22 degrees to artesian water at the same temperature has been determined at 1.02610, a value probably within .00005 for sea water collected from the surface at the shore or from ships. Second, the effects of various assumptions in regard to the conditions and temperatures of balancing columns of sea water are shown. Finally, whether one uses for the Ghyben-Herzberg ratio the value 38.3 based on the assumption of ground temperatures for both salt and fresh water, or the value 38 as the whole number giving least error, the estimate of the depth of the zone of transition for any given well is considerably reduced from those obtained using the ratios 40 or 42. In the case of the Beretania area of Honolulu, with a 30-foot head assumed, the estimated depth of this zone is 1,280 feet, 1,200 feet, 1,149 feet, or 1,120 feet, according as ratios 42, 40, 38.3 or 38 are used. The concept that the zone of diffusion is at least 50 feet and possibly 80 feet closer to the bottom of the wells in this area than has previously been estimated is of distinct practical importance, recognition of which, in the event of extreme reduction of head, might become critical.