STUDIES ON THE VALIDITY OF DARCY'S LAW
FOR FLOW IN NATURAL SANDS

Robert E. Carver

DEPARTMENT OF GEOLOGY
UNIVERSITY OF GEORGIA
ATHENS, GEORGIA 30602

in cooperation with the
ENVIRONMENTAL RESOURCES CENTER
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ATLANTA, GEORGIA 30332
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Technical Completion Report
USDI/ONRR Project No. A-037-GA
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REPORT DESCRIPTION

This Technical Completion Report is submitted to the Office of Water Resources Research in fulfillment of the reporting requirements for USD/OWRR project A-037-GA, titled "The non-linear flow component of well entrance losses".

AUTHOR AND PRINCIPAL INVESTIGATOR

Robert E. Carver is Associate Professor of Geology and Assistant Department Head in the Department of Geology, University of Georgia, Athens, Georgia. He was the Principal Investigator on the USD/OWRR sponsored project A-037-GA which was the basis of the research for this report.

ACKNOWLEDGEMENTS

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The project was conceived as a pilot study to develop techniques and establish ranges of variables necessary for a more extensive investigation of the role of inertial and frictional forces in "well entrance losses": the difference between mathematically predicted and actual well capacities. Well entrance losses are commonly assumed to be due, in part, to non-Darcy flow near the well, but concrete evidence for non-Darcy flow of water in natural sands at any reasonable macroscopic flow velocity does not appear to exist.

Attempts to standardize procedures for determining pressure and flow velocity relationships at low flow velocities, that is to obtain consistent base-line permeabilities for the samples to be tested, failed repeatedly. Ultimately it was demonstrated that, at low flow velocities, permeability of sands varies with macroscopic flow velocity in a very complex way, and that some earlier studies purporting to support the validity of Darcy's Law at very low flow velocities actually support this view.

With increasing flow velocity, beginning with velocities below 0.005 cm/sec, permeability increases rapidly in the α intergranular flow regime, decreases rapidly through the β flow regime, increases relatively slowly with increasing velocity in the γ flow regime, then remains nearly constant or decreases in the δ flow regime. The changes in flow resistance appear to be related, in part, to the surface chemistry of quartz.

Experiments at high flow velocities showed no dramatic change in permeability in the range of 1 to 2 cm/sec, but, because of problems with piezometry, the results are not considered conclusive.

On the basis of the research completed in this pilot study, one basic cause of "well entrance losses" probably is misestimation of the average permeability of the aquifer, either because gas permeameters are used, or because tests with water are conducted in the first three flow regimes.
1. INTRODUCTION

1.1. Review of Previous Work

Despite the fact that methods for the analysis and prediction of water-well capacities are fully developed and mathematically sophisticated, large errors in the prediction of capacities are the rule, rather than the exception. The errors are almost invariably in the direction of overestimation and the differences between real and predicted capacities are commonly ascribed to "well entrance losses". Well entrance losses are assumed to be due to improper well completion and to inertial and turbulent flow in the immediate vicinity of the well.

It has long been recognized that the fundamental law of fluid flow through porous media, Darcy's Law, does not hold true for flow velocities at which either appreciable inertial forces or turbulent flow occur (Darcy, 1856; Hubbert, 1940). Surprisingly, very little of the vast literature on the flow of fluids through porous media is devoted to the problem non-Darcy flow, probably because there are few natural or industrial situations in which deviations from Darcy's Law are anticipated to be of consequence, and because contemplation of the failure of an important and mathematically elegant physical law is repugnant to most physical scientists. Further, in most of the relatively few studies of non-linear flow in porous media, the porous media have been patently atypical of materials in which water wells are most commonly developed, partly because it is so difficult to produce high macroscopic flow velocities in fine- to medium-grained, or even coarse-grained natural sands. Therefore, the data and theory required to determine what proportion of well entrance losses are due to non-linear flow near the well are not available.
Lindquist (1933) investigated high-velocity flow of water in large brass tubes filled with lead shot and found that Darcy's Law is valid for flow conditions in which \( R_d \) is less than 4. \( R_d \) is a Reynolds number defined as

\[
R_d = \frac{Ud}{v}
\]

(1)

where \( U \) is the macroscopic, or discharge velocity; \( d \) is the diameter of the lead shot; and \( v \) the kinematic viscosity. For values of \( R_d \) greater than 4, Lindquist found the relation

\[
\frac{dh}{dl} = aU + bu^2
\]

(2)

in which \( dh/dl \) is the pressure loss per unit length and \( a \) and \( b \) are constants complexly related to size, shape, roughness and other factors for a given porous medium, to be valid.

Equation 2, with minor modifications, appears repeatedly in the literature on non-linear, or mixed, flow (Forshheimer, 1901; Muskat, 1949, p. 128; Englund, 1953, p. 14; Ward, 1964, p. 10, et al.), probably more because of its nice progression of mathematical terms, and its simplicity, than its applicability to the physical systems involved. None the less, the equation appears to be basically valid, at least for flow in the laminar-inertial regime.

Wycoff (et al., 1933) discussed methods of measuring permeability and found significant deviations from Darcy flow at velocities above 0.4 cm/sec for one sample and 0.7 cm/sec for a second sample. The samples are labeled "Filtros disk grade R" and "Filtros disk grade H" and one would suppose that they were commercial sands with relatively restricted ranges of grain size. Based on data from these two samples they concluded that "...viscous flow would obtain up to relatively high pressure-gradients and that only in the
immediate vicinity of a well flowing at high rates would liquids depart seriously from viscous flow." In spite of the negative phrasing, this statement neatly pinpoints the problem of well entrance losses. Incidentally, Hubbert's (1940) discussion of the role of non-Darcy flow is obviously based on the Wycoff paper.

Wycoff (et al., 1933) obtained very good correlation between liquid and gas permeabilities in five out of six tests. Their assertion that gas is preferable to liquids for permeability determination (p. 401-402) is one reason for the general acceptance of the gas method. Their results must have been purely fortuitous because this and other studies (see Section 2.2) have shown very low correspondence between gas and water permeabilities.

Fancher, Lewis, and Barnes (1933), plotted friction factor vs. Reynolds number over long ranges for a number of materials, and found apparent deviations from Darcy flow at Reynold's numbers of 1 to 10 for samples of Woodbine Sand and other sands. Unfortunately, Fancher, Lewis, and Barnes were primarily interested in the relationship between permeability and porosity and methods of measuring both properties. The discussion of work previous to 1933 is excellent, but little attention was devoted to deviations from Darcy's Law and certain critical data were omitted (i.e. the average grain size). However, it is clear from their Fig. 6 that deviations from Darcy's Law occurred, in two Woodbine Sand samples, at velocities as low as 0.4 cm/sec.

Englund (1953), experimenting with granules \(d = 0.26 \text{ cm}\) and very coarse sand \(d = 0.14 \text{ cm}\) described as "flinty and calcareous" and therefore probably quite angular and poorly sorted, found equation 2 to be valid for flows up to \(R_d = 150\). Englund's values for \(a\) and \(b\) were nearly twice the values found by Lindquist (1933) for lead shot, as might be expected because
of the greater angularity and roughness of Englund's media. Englund's results were linear over long ranges in experiments in which the only significant variables were the macroscopic flow velocity \( R_d = Ud/\nu \) and velocity-dependent permeability \( (1g_d^2/\nu) \), where \( I = U/K \). It is, therefore, doubtful that fully turbulent flow was established in this series of experiments.

Anandakrishnan and Varadarajulu (1963) tested four samples of sand, using an air-pressured permeameter similar to the one used in the high velocity studies reported here. The sand samples are described in terms of the range of sieve sizes and the tenth percentile of size distributions ("effective grain size" of civil engineering practice), but the source and character of the sand samples are not further described. However, "sand fractions" are mentioned in the discussion of constant lead test data and it is probable that the samples represented synthetic mixes of sieve fractions from some single sand sample. In the coarse and medium sands, departures from Darcy flow occurred at all velocities, from about 0.01 to 1.1 cm/sec. For the fine sand, deviations from Darcy's Law began suddenly at velocities near 0.1 cm/sec, and in the case of very fine sand flow was essentially linear up to velocities of 1.2 cm/sec. On the basis of this study, non-linear flow is more prevalent, and begins at lower velocity in coarse sands than in fine sands, but the relationships have not been quantified or related to permeameters of natural sands.

Ward (1964) found the equation

\[
\frac{dh}{dl} = \frac{\nu U}{K} + 0.550 \frac{c U^2}{K^2}
\]

(3)

to be valid for a variety of materials; including glass beads, crushed anthracite, gravel and medium to coarse sand; and concluded that the equation was valid for any material and both laminar and turbulent flow.

Lindquist (1965), in a discussion of Ward's paper, demonstrated that there was not enough information that infers that the equation could be expressed as

\[
U = \frac{c U^2}{K^2}
\]

using only data derived from macroscopic flow velocity measurements.

The data would be considered a constant lead test if it were provided at constant pressure. With this opinion, the data are unexplained, and the exponent of the equation is interpreted for sand.
was no significant difference between his earlier results and Ward's; that is turbulent flow had not been achieved. Several other discussants expressed doubt as to the universality of the numerical constant in equation 3, above.

Unfortunately for their application to well entrance losses, Ward's data do not cover a very wide range for any given material. High macroscopic flow velocities were achieved in coarse materials, but not low velocities; low velocities in fine-grained materials, but not high velocities.

These few papers appear to represent the major part of work that could be considered to have a significant bearing on the problem of non-Darcy flow as a cause of well entrance losses, and the fact is that they do not provide us with much that is of any practical use. It is clear that opinion favors either a Forshheimer equation (2) with an exponent of 2, or less, or a Fanning friction factor, but we have no reliable guide to what exponent or what friction factor would be appropriate for a fine-grained sand.

1.2. Objectives

Because none of the studies discussed or cited above provide data on the high-velocity flow of water in natural fine- to medium-grained sand, it was not, at the outset, possible to predict how much "well entrance loss" might be due to non-linear flow effects, as opposed to improper completion of the well, or other purely mechanical factors. The original objectives of the study were, therefore, to gather sufficient experimental data on natural sands in the most common size ranges to permit engineering estimates of the non-linear flow component of well entrance losses, if any, and ultimately to provide a basis of estimation based on grain size distribution alone.
As a model for the range of macroscopic flow velocities to be studied, an 8 inch diameter well, with 20 feet of screen set at 300 feet, producing 2,000 gallons per minute (Siple, 1967, Well No. 21F) was considered. For this well the macroscopic entrance velocity would be 1.6 cm/sec, the intergranular flow velocity 4.8 cm/sec, or more, and the macroscopic flow velocity 300 m from the well about 0.001 cm/sec.

It was considered desirable to begin with relatively coarse sands and to work toward finer sands, and to begin with low velocities and work toward high velocities, with the ultimate objective of achieving flow velocities of around 2 cm/sec in both coarse and fine sands. This procedure would provide the maximum opportunity to check the equipment for internal resistance, or other aberrations, and to correlate my data with the results of previous studies.

If data on departure from Darcy's Law, due to either inertial or turbulent effects, for a sufficiently large number of natural sand samples could be obtained, it was expected that the degree of departure, for any given velocity, could be predicted from grain size distribution parameters alone. Krumbein and Monk (1941) have shown that permeability can be estimated from a combination of mean diameter and standard deviation and Chilingar (1964) has shown that permeability can be roughly estimated from the porosity and mean grain size. If permeability can be estimated from these much more easily measured parameters, the degree of departure from Darcy's Law, or the range of applicability and the constants for equation 2, should be predictable from the same type of data.

Ultimately, given a sample of the aquifer sand, we could determine, in less than an hour, the adjustment of calculated well capacity that would be required to allow for non-Darcy flow in the immediate vicinity.
of the well. This was the long-range objective and it was not expected that it would be accomplished within the term of Allotment Project A-037-GA.

The immediate objectives were to develop a technique to produce relatively high velocity flows in medium- and fine-grained sands, to carry experiments out to at least the 1.6 cm/sec velocity mentioned above, and to assess the problem.
2. PROCEDURES

2.1. Samples

Large bulk samples (approximately 30 pounds per sample) of sands of various grain sizes, from a variety of depositional environments, were collected for the initial phase of the study. In all, ten samples were collected and three others were selected from bulk samples previously collected. For reasons explained in Section 3.1., some samples were modified by removing, by wet sieving, all material finer than 3.75Ø, or all material finer than 3Ø. In this case the original, unmodified sample was renumbered, with an "A" following the sample number. The sample modified by removing the material finer than 3.75Ø was indicated by a "B" following the sample number and the sample modified by removing the material finer than 3Ø by a "C" following the sample number.

Descriptions of sample localities are presented in Appendix B and the geological characterization of samples is summarized in Table 1.

2.2. Analysis of Samples

The large raw samples were dried under infrared lamps and split, through a riffle type sample splitter, to quantities suitable for the tests to be performed. Representative fractions of all samples, except 11 through 13, were sieved at ½Ø intervals through six-inch diameter brass sieves and moment statistics were computed by the method of Collias (et al., 1963). Data for samples 11 through 13 were taken from a previous study and are based on a 1Ø sieve interval. Results of sieve analysis are presented in Table 2.

Dry nitrogen permeability (Kg) of samples was determined by modifying a Ruska gas permeameter to accept a sample holder 100 cm long and 2.5 cm in diameter. Gas permeability, compared with permeability to water as determined
Table 1. Geological Characterization of Samples.

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Probable Environment of Deposition</th>
<th>Probable Geologic Age</th>
<th>Geologic Formation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fluviatile</td>
<td>Late Cretaceous to Early Eocene</td>
<td>Middendorf Fm.</td>
</tr>
<tr>
<td>2</td>
<td>Fluviatile</td>
<td>Pleistocene</td>
<td>Barnwell Sand</td>
</tr>
<tr>
<td>3</td>
<td>Fluviatile</td>
<td>Late Eocene</td>
<td>Barnwell Sand</td>
</tr>
<tr>
<td>4</td>
<td>Deltaic</td>
<td>Late Eocene</td>
<td>Barnwell Sand</td>
</tr>
<tr>
<td>5</td>
<td>Deltaic</td>
<td>Late Eocene</td>
<td>Barnwell Sand</td>
</tr>
<tr>
<td>6</td>
<td>Natural Levee</td>
<td>Holocene</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Point Bar</td>
<td>Holocene</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Shallow Marine</td>
<td>Late Eocene</td>
<td>Twiggs Clay</td>
</tr>
<tr>
<td>9</td>
<td>Fluviatile¹</td>
<td>Late Cretaceous</td>
<td>Middendorf Fm.</td>
</tr>
<tr>
<td>10</td>
<td>Fluviatile</td>
<td>Late Cretaceous</td>
<td>Middendorf Fm.</td>
</tr>
<tr>
<td>11</td>
<td>Beach</td>
<td>Holocene</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Beach</td>
<td>Holocene</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Dune</td>
<td>Holocene</td>
<td></td>
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¹Commercial sand, washed in rake classifier before collection.
Table 2. Results of Sieve Analysis of Samples.

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Mean Size</th>
<th>Standard Deviation</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Percent Smaller Than 4 Ø</th>
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<tr>
<td>1A</td>
<td>-0.8 Ø</td>
<td>1.28 Ø</td>
<td>-0.02 Ø</td>
<td>2.46 Ø</td>
<td>0.08</td>
</tr>
<tr>
<td>1B</td>
<td>-0.8</td>
<td>1.28</td>
<td>-0.02</td>
<td>2.38</td>
<td>0</td>
</tr>
<tr>
<td>1C</td>
<td>-0.9</td>
<td>1.26</td>
<td>-0.07</td>
<td>2.30</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1.98</td>
<td>0.87</td>
<td>-0.96</td>
<td>7.62</td>
<td>2.36</td>
</tr>
<tr>
<td>3A</td>
<td>0.50</td>
<td>1.38</td>
<td>-0.18</td>
<td>4.05</td>
<td>2.21</td>
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<tr>
<td>3B</td>
<td>0.40</td>
<td>1.26</td>
<td>-0.63</td>
<td>3.73</td>
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<tr>
<td>4</td>
<td>1.64</td>
<td>0.66</td>
<td>0.12</td>
<td>3.10</td>
<td>0.07</td>
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<tr>
<td>5A</td>
<td>1.92</td>
<td>0.71</td>
<td>0.12</td>
<td>3.50</td>
<td>0.88</td>
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<tr>
<td>5B</td>
<td>1.89</td>
<td>0.68</td>
<td>-0.17</td>
<td>2.90</td>
<td>0</td>
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<tr>
<td>6</td>
<td>2.24</td>
<td>0.72</td>
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<td>3.54</td>
<td>3.04</td>
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<tr>
<td>7</td>
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<td>0.95</td>
<td>0.37</td>
<td>3.16</td>
<td>0.52</td>
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<tr>
<td>8</td>
<td>0.82</td>
<td>0.82</td>
<td>0.17</td>
<td>4.82</td>
<td>0.44</td>
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<tr>
<td>9</td>
<td>0.16</td>
<td>1.08</td>
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<tr>
<td>10A</td>
<td>1.26</td>
<td>1.00</td>
<td>0.34</td>
<td>3.40</td>
<td>1.52</td>
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<tr>
<td>11</td>
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<td>1.13</td>
<td>-0.31</td>
<td>1.90</td>
<td>0.10</td>
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<tr>
<td>12</td>
<td>1.93</td>
<td>1.14</td>
<td>-0.81</td>
<td>2.71</td>
<td>0.21</td>
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<tr>
<td>13</td>
<td>2.43</td>
<td>0.47</td>
<td>-0.16</td>
<td>4.67</td>
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Table 3. Comparison of Gas and Water Permeabilities.

<table>
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<tr>
<th>Sample Number</th>
<th>Dry Nitrogen Permeability, Kg x 10^8 cm^2</th>
<th>Water Permeability, K x 10^8 cm^2</th>
<th>K x 100 Kg</th>
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<td>133</td>
<td>84</td>
<td>63</td>
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<td>16</td>
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<td>7</td>
<td>61</td>
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<td>36</td>
</tr>
<tr>
<td>8</td>
<td>Very High</td>
<td>7</td>
<td>-</td>
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<td>9</td>
<td>Very High</td>
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<td>-</td>
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</tbody>
</table>
in low flow velocity experiments, is presented in Table 3. The data shown in Table 3 confirm von Engelhardt and Tunn's (1954) observation that gas permeabilities are very much higher than water permeabilities, and show that the relation is true for unconsolidated sands of relatively high permeability, as well as for the low permeability sandstones examined by von Engelhardt and Tunn. Further, the relationship between Kg and K is quite inconsistent. In this series of experiments K ranged from 10 to 79 percent of Kg. Predictions of well capacity based on air permeabilities should therefore be expected to be much higher than the true well capacity, and higher by an unpredictable amount.

2.3. Procedure for Low Flow Velocity Experiments

For low macroscopic flow velocity tests an overflow type constant head device was employed. The constant head device consisted of an inverted aspirator bottle in which the side tubulature served as the inlet port. The overflow tube, an air pressure compensation tube, and the outlet tube to the permeameter entered the aspirator bottle through a three-hole rubber stopper. The inverted aspirator bottle was supported in a rope sling and could be raised and lowered, by means of a rope passing through a pulley (block) attached to the ceiling, from a point below the permeameter to a point approximately 2.5 m above the centerline of the permeameter.

The permeameter consisted of a polycarbonate tube, 7.35 cm in inside diameter and 24 cm long, in horizontal position. At each end of the tube was a one-hole stopper, a 3/4-inch thick rubber washer, two layers of 1/4-inch mesh galvanized steel screen and a final 120 mesh stainless steel screen. The stopper assemblies were held in the tube, when under pressure, with pipe clamps. Pressure ports 1/16-inch in diameter and internally screened with 120 mesh stainless steel screen cloth were drilled 7, 12,
and 17 cm from the inlet end of the permeameter. The pressure ports were externally connected to manometer tubes filled with water.

Samples were tamped into the permeameter in layers 2 to 5 cm thick, while thoroughly wet, and water was run through the sample in the permeameter, at maximum pressure, for at least an hour before actual tests were begun. The test schedule always began with high pressures, proceeding to low pressures, and in most cases back up to high pressures, so that any change in permeability due to growth of bacteria, saturation of the sample, etc. would be detected on analysis of the data. Tap water was used in all permeability tests, partly because tap water is more like ground water than distilled water is, and partly because the chlorination would prevent growth of bacteria in the sample.

The pressure differential along the length of the permeameter was adjusted by raising the constant-head device to some predetermined height above the permeameter. The pressure difference, to the nearest 1/8 inch was determined, and the time required for 1 liter of water (less in the case of very low flow velocities) to flow through the sample was measured, with a stopwatch, to the nearest second. The test was then repeated and the times required for 1 liter of flow were averaged. Temperature of the water in the receiving cylinder was measured immediately after each test.

Tests with the permeameter loaded with lead shot 2 mm in diameter, at maximum pressure for this apparatus, yielded no measurable difference in head along the length of the permeameter and it was concluded that frictional losses in the permeameter itself were negligible. It was, therefore, surprising and vexatious that low velocity flow determinations of permeability were not consistent, as discussed in section 3.1. Ranges of pressures and flow velocities achieved in low pressure experiments are summarized in Table 4.
Table 4. Ranges of Variables, Low Velocity Experiments.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure Difference</td>
<td>0.5 cm H₂O</td>
<td>170 cm H₂O</td>
</tr>
<tr>
<td>Velocity</td>
<td>0.0013 cm/sec</td>
<td>1.4 cm/sec</td>
</tr>
<tr>
<td>Permeability</td>
<td>2 x 10⁻⁸ cm²</td>
<td>237 x 10⁻⁸ cm²</td>
</tr>
</tbody>
</table>

2.4. Procedure for High Flow Velocity Experiments

For high flow velocity studies a gas pressurized water reservoir was used. A six foot length of six-inch diameter steel pipe was welded closed at the bottom end and tapped for a 1/2-inch nipple and valve just above the closure. A flat cap with three 1/2-inch valves was flanged to the upper end. The lower valve served as the outlet to the permeameter. The upper valves were the air escape and blowdown valve, the filling valve, and the pressure line valve. The pressure line was coupled, through a pressure reducing valve, to a nitrogen tank, the filling line to a tap water source. Water level in the reservoir was indicated by five neon bulbs connected to a copper rod suspended along the center line of the reservoir and to five copper rods inserted through, and insulated from, the wall of the reservoir at progressive distances from the base.

The high pressure permeameter was constructed from a 60 cm length of four-inch diameter steel pipe threaded for standard pipe caps. The caps were drilled and tapped for 1/2-inch nipples, the inlet and outlet pipes. At each end of the pipe were a 1/2-inch thick rubber washer, an 18 cm long tubular spacer, a perforated stainless steel plate, and a 120 mesh stainless steel screen.
The basic design of the permeameter was satisfactory from a hydro-
dynamic standpoint, but was mechanically awkward, and difficult to assemble
to a water-tight condition. It is not recommended for further work.

Pressure ports consisted of 3/8-inch brass nipples tapped through the
wall of the permeameter 20, 30 and 40 cm from the upstream end. Wooden
plugs, with 1/16-inch diameter holes screened at the inside end with 120
mesh stainless steel wire cloth, were fitted into the nipples, flush with
the inner wall of the permeameter, and cemented in place. Permeameter
pressures were measured with mercury manometers at relatively low pressures
(up to about 450 mm) and mechanical gauges at higher pressures. In static
tests the mechanical gauges read 10% high, relative to mercury manometers
and all computations were corrected for this difference.

The procedures for packing samples in the permeameter, running tests
(high to low pressures and back to high pressures), and measuring temperatures
were identical to the low pressure series of tests, except that times for
transmission of 10 to 20 liter samples were measured.

2.5. Computations

Permeability was computed in \( \text{cm}^2 \) units as in the following formula
adapted from Curtis (1971):

\[
K = \frac{Q \mu l}{A \rho gh} \tag{4}
\]

in which,

\( K \) = permeability, in \( \text{cm}^2 \)

\( Q \) = volume of flow, in \( \text{cm}^3/\text{sec} \)

\( A \) = cross-sectional area of permeameter, in \( \text{cm}^2 \)

\( \mu \) = viscosity in poise, gm sec/cm, or dyne sec/cm\(^2 \)
\( \rho = \text{mass density of water, in } \text{gm/cm}^3 \)

\( g = \text{acceleration of gravity, in cm/sec}^2 \) (979.57 cm/sec^2)

\( l = \text{length between first and last pressure port, in cm} \)

\( h = \text{difference in pressure between first and last port, in cm } \text{H}_2\text{O} \)

also

\[
\frac{Q}{A} = \bar{U} = \frac{V}{TA}
\]  

(5)

where

\( \bar{U} = \text{macroscopic flow velocity, in cm/sec} \)

\( V = \text{volume of water measured for each test run, in cm}^3 \)

\( T = \text{time required for } V \text{ to flow through the permeameter, in sec} \)

\( A = \text{as in equation 4} \)

Therefore, equation 4 can be rewritten

\[
K = \frac{V \bar{U} l}{T A \rho g h}
\]  

(6)

If we combine all factors that can be considered constant for a given set of experimental conditions, including constant temperature, that is \( V, \mu, l, A, \rho, \) and \( g, \) equation 6 may be written

\[
K = \frac{c}{Th}
\]  

(7)

In actual practice the water temperature varied to some extent (much more in the high velocity tests than in the low velocity tests) and the following equations were employed:

\[
K = \frac{\mu c'}{Th}
\]  

(8)

for the low velocity series, and

\[
K = \frac{\mu}{\rho} \cdot \frac{c''}{Th}
\]  

(9)
for the high velocity tests. Tables for $c'\text{ and } u/c''$ constructed for all temperatures encountered in the experimental work greatly facilitated the computational work. Values of the constant factors and of $c'$ and $c''$ are given in Table 5.

Table 5. Permeameter Constants.

<table>
<thead>
<tr>
<th>Constant, or Fixed Variable</th>
<th>Low Velocity Permeameter</th>
<th>High Velocity Permeameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi$</td>
<td>1,000 cm$^3$</td>
<td>10,000 cm$^3$</td>
</tr>
<tr>
<td>1</td>
<td>10 cm</td>
<td>20.1 cm</td>
</tr>
<tr>
<td>A</td>
<td>42.43 cm$^2$</td>
<td>81.71 cm$^2$</td>
</tr>
<tr>
<td>$g$</td>
<td>979.57 cm/sec$^2$</td>
<td>979.57 cm/sec$^2$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.997 gm/cm$^3$</td>
<td></td>
</tr>
<tr>
<td>$c'$</td>
<td>0.0950</td>
<td></td>
</tr>
<tr>
<td>$c''$</td>
<td></td>
<td>2.511</td>
</tr>
</tbody>
</table>
3. EXPERIMENTS AT LOW MACROSCOPIC FLOW VELOCITIES

3.1. Results of Experiments at Low Velocities

Initial experiments on the permeability of natural sand samples, run from high settings of the constant-head device to progressively lower settings, or from high to low and back to high, invariably failed to yield consistent results, in terms of Darcy's Law, at very low flow velocities. It was first suspected that design of the equipment was faulty, or that there were errors in procedure. Repeated modifications in the design of the permeameter, accompanied by blank and static tests of the equipment failed to either eliminate the discrepancy, or explain its cause.

Because it has been demonstrated that Darcy's Law does not hold for the flow of weak electrolytes through soils (Lutz and Kemper, 1959) attention was ultimately focused on the clay content of samples, even though the clay content of most samples was considered negligible (see Table 2). A split of sample 1, now designated sample 1A, was obtained and washed over a 200 mesh (3.75Ø) stainless steel sieve to remove all silt and clay. The washed sample, designated 1B, was tested for permeability at various macroscopic flow velocities. The permeability vs. flow-velocity curve was similar to the curve for the untreated sample, but permeability values were higher at all flow velocities, as shown in Fig. 1. Although the finer than 3.75Ø material had a significant effect on the overall permeability of the sample, it did not appear to be the basic cause of deviation from Darcy's Law at low flow velocities.

An additional split of sample 1 was obtained and all material finer than 120 mesh (3Ø) was removed by wet sieving. The modified sample was designated 1C and tested over a long range of flow velocities. The apparent-permeability vs. velocity curve was similar in shape to the curves for 1A
Figure 1. Permeability vs. Macroscopic Velocity.
and 1B, but permeability values were higher over the whole range of flow velocities and the low velocity drop in permeability appears much accentuated in the curve for 1C. Again, the finer sediment fractions do not seem to be the cause of deviations from Darcy's Law, they seem to dampen out the large deviation in permeability seen in the curve for sample 1C.

Similar curves of the relation between apparent permeability and flow velocity were obtained for ten other samples with progressively lower permeabilities, as shown in Figures 1 through 3. It therefore appears that most natural sands, to the extent that the samples tested represent natural sands in general, are much more permeable at very low flow velocities than at higher velocities, although it is difficult to see any direct relationship between magnitudes of the two variables.

Even more surprising was the relationship between apparent permeability and flow velocity for the two least permeable samples of the group, as shown in Fig. 4. Permeability was near average at extremely low velocities, increased as flow velocity increased, then decreased again. The readily accessible literature on the subject of permeability of sand-sized materials contained no hint that the observed effects would occur, and suggested that similar experiments had shown that they did not occur.

3.2. Review of the Literature Relative to Low Velocity Experiments

Swartzendruber in two papers (1962a, 1962b) reviewed the problem of application of Darcy's Law to very low flow velocities. He noted that King (1898) had found small deviations from Darcy's Law at very low flow velocities in sands. King's findings were largely ignored in subsequent years, and further studies of low flow velocity appear to have been motivated by Miller-Brownlie's (1919) report that in subsoils of Punjab, India no
Figure 4. Permeability vs. Macroscopic Velocity

\( \nu \times 10^{-3} \text{ cm/sec} \)

\( K \times 10^{-8} \text{ cm}^2 \)
detectable ground-water motion occurred at hydraulic gradients of less than 20 feet per mile. Between 1923 and 1934 an extensive study of permeabilities at low flow velocities was conducted under the direction of O. E. Meinzer and reported by Stearns (1928), Meinzer and Fishel (1934) and Fishel (1935). All of the reports appear to prove that Darcy's Law is valid at very low flow velocities.

Swartzendruber (1962b) reexamined the data reported by Stearns (1927), concluded that the data did indicate some deviation from Darcy's Law and that the deviations were due to clay content of the samples tested.

Meinzer and Fishel (1934) and Fishel (1935) extended the work done by Stearns to very low flow velocities. They employed a non-discharging type of permeameter, with a U-tube approximately 200 cm long, to obtain very low pressure gradients and flow velocities. Unfortunately, Meinzer and Fishel (1934) were unable to obtain consistent values of permeability for the sample tested, the same sample tested by Stearns (1928), even when the data were, in effect, averaged over four readings. In the two tests (of eight) reported, the variation in apparent permeability was 41 percent and 17 percent. Fishel did not average his data and the variation in observed permeability for the two tests (of six) reported was 133 percent in the first test and, taking the second lowest permeability value, 295 percent in the second test.

Further analysis of the data of Meinzer and Fishel (1934) and Fishel (1935) is not justified because falling head permeameters tend to average out any real variation in permeability with change in flow velocity and because the gross permeability of the sample changed, partly as a result of compaction of the sample, partly for reasons unknown during the course of the tests.
In 1954, von Engelhardt and Tunn reported a very interesting and important series of experiments. Working with cylinders cut from sandstone cores, they found that permeabilities to air, carbon tetrachloride, cyclohexane, and benzene were up to nine times greater than permeability to water, and commonly about four times greater. Permeability to NaCl solutions was consistently greater than permeability to distilled water. For one rock sample, permeability to the organic liquids was essentially constant through a tenfold increase in h/l, but permeability to water increased by a factor of five over the same range in hydraulic gradient. Similar increases in permeability to water were found for five other sandstone samples. Permeability to NaCl solutions also increased with increasing hydraulic gradient and fluid velocity.

Von Engelhardt and Tunn presented compelling arguments that the effects listed above were related to surface chemistry of the small quantities of clay (1 to 6 percent) present in the samples. However, they also presented evidence for surface activity of quartz and suggested (p. 16) that part of the drag effect may be due to hydrogen bonding of water on quartz.

Lutz and Kemper (1959) found that permeability of cation-saturated clays either increased, or decreased with increasing hydraulic gradient, but that permeability to electrolytes was almost invariably greater than permeability to water. A number of other papers, of later date, deal exclusively with the flow of water in clays, and are not discussed here.

3.3. Discussion of Experiments at Low Velocities

The experiments at low flow velocities performed during the course of this study were intended only to provide control for high velocity experiments that were the real objective. It was, therefore, disconcerting to
find large variations in the apparent permeability at low macroscopic flow velocities in samples that were considered to be virtually clay-free. Worse, the effect appeared to increase with attempts to remove clay-sized material in some cases (IA, B, C in Fig. 1) and to reverse in the case of the least permeable sands (Fig. 4). Much of the time and effort that would have been devoted to high flow velocity studies was diverted to repetition of low velocity experiments, with modifications of equipment and procedure, in attempts to eliminate these seemingly anomalous results.

The fact is that data reported by Stearns (1928) for extensive tests of a North Carolina beach sand reveal the same relation between flow velocity and permeability shown in Fig. 4. If the data for the 100 cm tube are grouped by quantity of flow and averaged (i.e. the 22 permeability values associated with flows of 1.50 to 2.49 mg/sec average to 400), and permeability is plotted against flow velocity, as in Plot A of Fig. 5, it is seen that permeability increases very rapidly at low velocities, then decreases and, finally increases again. Further, if the data for the 10 cm tube are ranked in order of increasing flow, averaged in groups of five and plotted (Fig. 5, Plot B) the same relation appears, but Plot B is shifted to the right relative to Plot A. Part of the shift, but not more than half of it, is due to the sequential averaging procedure applied to the data for Plot B, the remainder probably represents a difference in sample packing in the two tubes, as does the difference in overall permeability.

Data from Stearns (1928) therefore support my findings, particularly with respect to Samples 2 and 3 (Fig. 4). At very low flow velocities, here referred to as the $\alpha$ flow regime, resistance to flow decreases very rapidly with increase in flow velocity. At higher flow velocities, the $\beta$ flow regime, resistance increases rapidly with increasing flow velocity.
In plot a are the number of data averaged at each point.

Figures 3 and 4 are semi-logarithmic graphs, not part of the data set.

Figure 5: Permeability versus macroscopic velocity. Data of Stenn (1952), permeability in gallons per day per sq. ft. at hydraulic gradient of 1. The small numbers refer to the source of the data.
At still higher velocities, the \( \gamma \) flow regime, resistance to flow decreases slowly with increasing flow velocity; reaching, finally, the \( \delta \) flow regime in which resistance to flow increases very slowly, or remains constant with increasing flow velocity. Differences in resistance to flow in the \( \alpha \) and \( \delta \) flow regimes are not trivial, they are expressed as changes in permeability that commonly amount to 30 percent and may be as great as 50 percent.

The problem of the velocity ranges over which the \( \alpha \) through \( \delta \) flow regimes may be expected to occur is a formidable one because it appears that the ranges are inversely related to permeability, as indicated by terminations of the \( \delta \) regimes in Fig. 1, and because the ranges probably are further dependent on several complexly inter-related properties of the sands. However, based on the limited data available, the \( \alpha \) flow regime appears to terminate at flow velocities less than 0.005 cm/sec, and the \( \delta \) flow regime at velocities less than 0.025 cm/sec. The \( \gamma \) flow regime is much less clearly defined, indeed it does not appear at all on some plots, but it appears to terminate in the range between 0.05 and 0.3 cm/sec.

Unfortunately, samples studied by von Engelhardt and Tunn (1954) were much less permeable than the samples I studied, and the range in velocities from about 0.001 to about 0.04 cm/sec was covered in five or six steps. One might guess that most of von Engelhardt and Tunn's permeability data are from the \( \gamma \) flow regime, but there certainly is no way to corroborate this guess. Lutz and Kemper's (1959) materials were orders of magnitude less permeable than the materials employed in this study, and the true flow velocity can not be determined from their experimental data. It is interesting to note, however, that permeabilities of their samples either increased, or decreased over the 60 cm Hg pressure gradient employed in their experiments, suggesting that at least two different intergranular flow regimes are represented.
It is legitimate to speculate that at least two physiochemical effects, similar to the effects discussed by von Engelhardt and Tunn (1954) and Lutz and Kemper (1959), are involved in producing the observed variations in permeability. Based on the effects of removing clay and silt, and then removing silt and fine sand from Sample 1 (Fig. 1), it is probable that at least one of the physiochemical effects is not related to the surface activity of clay. This is supported by von Engelhardt and Tunn's observation of significant surface activity in quartz. Further, it is probable that inertial forces are responsible for decreases in permeability associated with flow regimes. The data from this study do not warrant further speculation; additional experiments similar to those of von Engelhardt and Tunn and Lutz and Kemper, but covering greater ranges of flow velocities and sediment sizes will be required to elucidate the mechanisms involved.
4. EXPERIMENTS AT HIGH MACROSCOPIC FLOW VELOCITIES

4.1. Results of Experiments at High Velocities

In the second, high flow velocity, phase of the study, every effort was directed toward achieving the basic objective of the project; a study of the validity of Darcy's Law, for fine- to medium-grained sands, at macroscopic flow velocities of 1.6 cm/sec and above. The ranges of pressure gradients and flow velocities achieved, with the range of observed permeabilities of the samples tested, are presented in Table 6. Equivalent data for the low velocity series of experiments is given in Table 4.

Table 6. Ranges of Variables, High Velocity Experiments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure Difference</td>
<td>35 cm H$_2$O</td>
<td>1440 cm H$_2$O</td>
</tr>
<tr>
<td>Velocity</td>
<td>0.03 cm/sec</td>
<td>2.3 cm/sec</td>
</tr>
<tr>
<td>Permeability</td>
<td>$2 \times 10^{-8}$ cm$^2$</td>
<td>$145 \times 10^{-8}$ cm$^2$</td>
</tr>
</tbody>
</table>

A list of samples with the range of velocities reached, the best estimate of permeability at the high end of the low velocity test, if available, and the best estimate of permeability at high velocity is presented in Table 7. Measured permeabilities were, in general, somewhat higher in the high velocity experiments than in the low velocity experiments, but were still surprisingly close, considering the difficulty of producing equivalent packing in permeameters of two entirely different designs.

Data for samples 1A, 1C, 8A, and 8C, typical of the entire high velocity series, are plotted in Figs. 6 and 7. There was no evidence of a
Table 7. Comparisons of Permeability at High and Low Velocities

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Range of Velocities</th>
<th>K, Low Velocity</th>
<th>K, High Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A</td>
<td>0.2 - 2.1 cm/sec</td>
<td>$75 \times 10^{-8}$ cm$^2$</td>
<td>$82 \times 10^{-8}$ cm$^2$</td>
</tr>
<tr>
<td>1C</td>
<td>0.2 - 2.3</td>
<td>126</td>
<td>108</td>
</tr>
<tr>
<td>2C</td>
<td>0.2 - 1.7</td>
<td></td>
<td>27</td>
</tr>
<tr>
<td>3C</td>
<td>0.2 - 2.2</td>
<td></td>
<td>70</td>
</tr>
<tr>
<td>4C</td>
<td>0.2 - 1.5</td>
<td></td>
<td>23</td>
</tr>
<tr>
<td>5C</td>
<td>0.2 - 1.5</td>
<td>18</td>
<td>21</td>
</tr>
<tr>
<td>6A</td>
<td>0.2 - 0.7</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>6C</td>
<td>0.1 - 1.6</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>8A</td>
<td>0.2 - 1.5</td>
<td>5</td>
<td>26</td>
</tr>
<tr>
<td>8C</td>
<td>0.3 - 2.3</td>
<td></td>
<td>12</td>
</tr>
</tbody>
</table>
Figure 6. Permeability vs. Macroscopic Velocity, High Velocity Experiments
Figure 7: Permeability vs. Macroscopic Velocity, High Velocity Experiments

\[ U \times 10^{-3} \text{ cm/sec} \]

\[ K \times 10^{-8} \text{ cm}^2 \]

Points A and C represent experimental data.
significant decline in permeability at the highest flow velocity for any sample, and the permeability of several samples appeared to increase slightly at higher velocities.

4.2. Discussion of Experiments at High Velocities

Unfortunately, it was necessary to assemble and construct the equipment for the high pressure, or high flow velocity, phase of the project during the latter part of the first year of the project and a period of the second year, July and August 1972, during which the project was not funded. Design and selection of the equipment was therefore governed more by expediency and economy than by my conception of the best way to do the job. In consequence, pressure control and piezometry became major problems.

The pressure reducing valve proved to be insensitive to small changes in pressure and to have a downstream pressure limit of 20 psi, far below specifications for the device. The pressure gauges proved to be quite inaccurate, again far below specifications, and attempts to recalibrate the gauges probably were not entirely successful. Additionally, the mercury manometers were not fully isolated from the permeameter, that is water was allowed to flow through the manometer ports and up the connecting tube, reducing the accuracy of the pressure differential measurements. In total, the problems of piezometry and pressure control resulted in relatively large random variations in apparent permeability of the samples tested. The results of the high flow velocity tests are, therefore, not considered conclusive, especially in view of the distinct flow regime changes in permeability observed in the low velocity series of tests (1B and 1C, Fig. 1; 4, Fig. 2).
Previous work, reviewed in the introduction to this report, indicates that deviations from Darcy's Law do occur at high flow velocities, but that deviations are very complexly related to physical character of the permeable media. It is probable that the results of the experiments reported here are essentially correct; that no large deviations from Darcy's Law occur at velocities between 0.2 and 2.0 cm/sec in most natural sands. However, the problem deserves a much more thorough investigation.
5. SUMMARY AND IMPLICATIONS OF THE RESEARCH

Experiments on natural and modified sand samples indicate that large departures from Darcy's Law occur at very low flow velocities. For the samples studied, there is an $\alpha$ flow regime in which permeabilities increase rapidly with increases in macroscopic flow velocity up to about 0.005 cm/sec, the difference amounting to as much as 30 to 50 percent of permeability at the lowest measured flow velocity. With increasing flow velocity, the $\alpha$ flow regime gives way to the $\beta$ flow regime in which permeabilities decline rapidly with increasing flow velocities, up to velocities of about 0.025 cm/sec, ultimately reaching approximately the same permeabilities observed at the low range of the $\beta$ flow regime. Beyond the $\beta$ flow regime permeability again increases, relatively slowly, up to velocities of between 0.05 and 0.3 cm/sec, but the increase is relatively small and does not occur at all in some sands. This third flow regime is designated the $\gamma$ macroscopic flow regime. On the basis of the experiments reported here, the last significant variation in permeability occurs in the velocity range above the upper limit of the $\gamma$ flow regime, where a gradual decline of permeability occurs (the $\delta$ flow regime). Examination of the data of Stearns (1928) confirms the results of this study and the existence of the $\gamma$ through $\delta$ macroscopic flow regimes.

No significant deviations from Darcy's Law were observed in the series of high velocity experiments (0.2 to 2.3 cm/sec), but the results are not considered conclusive, especially in view of the $\delta$ flow regime variations observed with the low velocity equipment.

It is concluded that laboratory tests of the permeability of porous media should be conducted in constant head permeameters, with water, over the range of macroscopic flow velocities that are most likely to apply to
the practical problem at hand. Data from falling-head permeameters tend to average-out significant variations in permeability, without revealing the exact character of the averaged variations, and may lead to conclusions that are not applicable to the natural condition. Single tests, at one flow velocity, may coincide with an abnormally high, or low, permeability in the \( \alpha \), \( \beta \), or \( \gamma \) flow regime and result in gross errors in estimates of overall permeability. Permeability estimates based on tests with dry gas are, in general, much too high and are poor predictors of permeability to water.

Field tests of permeability are subject to the same errors; they may erroneously average-out significant variations in permeability, or they may represent an anomalous permeability associated with a small range of flow velocities, depending on the original assumptions, observation well spacing, and other factors.

It appears probable that the major factor in "well entrance losses" is neither improper completion of wells, nor inertial and frictional flow effects near the well, but failure to recognize that intergranular flow is not directly proportional to hydraulic gradient at low values.
6. REFERENCES CITED


Engelund, Frank, 1953, On the laminar and turbulent flows of ground water through homogeneous sand: Academiet for de tekniske videnskaber, Kobenhavn, n. 3, 105 p.


APPENDIX A. KEY TO SYMBOLS

a, b = dimensionless coefficients

c, c', c" = coefficients consisting of combined fixed variables

A = cross-sectional area \(\text{cm}^2\)
d = diameter \(\text{cm}\)
g = acceleration of gravity \(\text{cm/sec}^2\)
h = head, pressure differential \(\text{cm} \ H_2O\)
K = permeability, water \(\text{cm}^2\)
K_g = permeability, gas \(\text{cm}^2\)
l = length \(\text{cm}\)
Q = discharge \(\text{cm}^3/\text{sec}\)

R_d = a Reynolds' number, Ud/\nu \ "dimensionless"
T = time for a given volume to flow \(\text{sec/volume}\)
U = macroscopic velocity \(\text{cm/sec}\)
V = volume \(\text{cm}^3\)
u = dynamic viscosity \(\text{dyne sec/cm}^2\) \(\text{gm/cm sec}\)
v = kinematic viscosity \(\nu/\rho\) \(\text{cm}^2/\text{sec}\)
\rho = mass density \(\text{gm/cm}^3\)
\phi = phi size transformation (Krumbein, 1938) \(1/2\phi = 10d \ \text{mm}\)
APPENDIX B. SAMPLE LOCATIONS.

Sample 1. From gravelly, very coarse, cross-bedded sand of the Middendorf Formation, formerly called the Tuscaloosa Formation, at the Windsor Spring section, along Windsor Spring Road 1 mile north of Tabacco Road, south of Augusta, Richmond County, Georgia. The section is further described in Sandy, Carver, and Crawford (1966, p. 13). The Middendorf Formation ranges in age from Late Cretaceous to Early Eocene and is considered to be predominantly fluviatile in origin.

Sample 2. Medium-grained sand from a sand pit cut in a terrace on the right bank of the Savannah River below U.S. Corps of Engineers Lock and Dam No. 9 below Augusta, Richmond County, Georgia. The terrace deposit is probably Pleistocene in age and the sands are fluviatile in origin.

Sample 3. Coarse-grained, cross-bedded, gravelley sand from an abandoned sand pit on the west side of the Augusta Concrete Block Company plant at the left valley wall of the Savannah River 0.5 miles northeast of the Georgia-South Carolina boundary on U.S. Highway 1, Aiken County, South Carolina. The sample is from the base of the Barnwell Formation, of probable Late Eocene age, as described in an earlier paper (Carver, 1972, Section 13, Unit 3), and probably is fluviatile in origin.

Sample 4. Medium-grained sand from a borrow pit on the north side of U.S. Highway 1, 0.5 miles east of Meadowbrook Drive on the southwest side of Augusta, Richmond County, Georgia. The sand probably is Late Eocene Barnwell Formation sand and probably represents a deltaic environment of deposition.
Sample 5. Medium-grained sand from a shallow cut on the north side of old U.S. Highway 1, south of the newer four-lane highway, and 0.6 miles northeast of the intersection of Highway 1 and Tobacco Road, southwest of Augusta, Richmond County, Georgia. Except for the somewhat finer grain size, the sample is identical to sample 4.

Sample 6. Fine-grained sand from a natural levee on the left bank of the Middle Oconee River, Forest Heights subdivision, Athens, Clarke County, Georgia. The natural levee is Holocene in age.

Sample 7. Medium-grained sand from a point bar on the left bank of the North Oconee River below the old mill dam 200 yards south of the Oconee Street bridge, Athens, Clarke County, Georgia. The point bar is Holocene in age.

Sample 8. Coarse, white, structureless sand from a shallow pit on the south side of Georgia Highway 57, 9.5 miles east of its intersection with U.S. Highway 80, east of Macon, Georgia and in Twiggs County, Georgia. At this locality opaline, montmorillonitic clays of the Twiggs Clay formation are interbedded with white to orange sands. The Twiggs Clay is of Late Eocene age and of shallow marine, or lagoonal origin. The sample locality was Stop 3 of the second day of the 1971 Georgia Geological Society field trip.

Sample 9. Coarse sand from a sand pit on the east side of U.S. Highway 80, 0.9 miles south of its intersection with Georgia Highway 57, east of Macon, Bibb County, Georgia. Sample 9 is classified sand (clay washed out) from the Middendorf Formation mined in this pit. The Middendorf Formation is Late Cretaceous to Early Eocene in age and probably fluviatile in origin.
Sample 10. Medium-grained, clayey sand from the Middendorf Formation across Donan Road and approximately 0.5 miles southeast of the locality of sample 9. The sample is from the top of a thick, cross-beded sand unit below a kaolin lens. The age and environment are the same as for sample 9.

Sample 11. Medium-grained beach sand from Huntington Beach State Park, South Carolina. The sample was taken from the top of a low bar 6 yards out from the last high-tide line. The beach is Holocene in age.

Sample 12. Medium-grained beach sand from Huntington Beach State Park. The sample was taken 31 yards from the last high-tide mark.

Sample 13. Fine-grained dune sand from Huntington Beach State Park. The sample location was in sea oats 10 yards behind the first fixed dune ridge. The dunes are Holocene in age.