

THE
**HAWAII GEOTHERMAL
PROJECT**

BUOYANCY INDUCED FLOWS IN A SATURATED
POROUS MEDIUM ADJACENT TO
IMPERMEABLE HORIZONTAL SURFACES

TECHNICAL REPORT No. 12



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NOMENCLATURE

A	constant defined by Eq. (6b)
C	specific heat of the convective fluid
f	dimensionless stream function defined by Eq. (16)
g	acceleration due to gravity
h	local heat transfer coefficient
\bar{h}	average heat transfer coefficient defined by Eq. (33)
K	permeability of the porous medium
k	thermal conductivity of the porous medium
L	length of the heating or cooling surface
Nu_x	local Nusselt number, $Nu_x = hx/k$
\bar{Nu}	average Nusselt number, $\bar{Nu} = \bar{h}L/k$
p	pressure
Q	over-all heat transfer rate
q	local heat transfer rate
Ra	modified Rayleigh number, $Ra \equiv \overline{T_w - T_\infty} \rho_\infty g \beta K L / \mu \alpha$
Ra_x	modified local Rayleigh number, $Ra_x \equiv \rho_\infty g \beta K T_w - T_\infty x / \mu \alpha$
S	spanwise dimension
T	temperature
u	velocity component in x-direction
v	velocity component in y-direction
x	horizontal coordinate
y	vertical coordinate
α	equivalent thermal diffusivity
β	coefficient of thermal expansion

δ_m	momentum boundary layer thickness
δ_T	thermal boundary layer thickness
$\bar{\eta}$	dimensionless similarity variable defined by Eq. (15)
η_m	value of η at the edge of momentum boundary layer
η_T	value of η at the edge of thermal boundary layer
θ	dimensionless temperature defined by Eq. (17)
λ	constant defined by Eq. (6b)
μ	viscosity of convective fluid
ρ	density of convective fluid
ψ	stream function

Subscript

∞	condition at infinity
w	condition at the wall

ABSTRACT

Boundary-layer analysis is performed for the buoyancy-induced flows in a saturated porous medium adjacent to horizontal impermeable surfaces. Similarity solutions are obtained for the convective flow above a heated surface or below a cooled surface, where wall temperature is a power function of distance from the origin. Analytical expressions for boundary layer thickness, local and overall surface heat flux are obtained. Applications to convective flow in a liquid-dominated geothermal reservoir are discussed.

I. Introduction

It is well known that if the temperature of a horizontal surface differs from that of the surrounding fluid, a vertical density gradient will be generated in the surrounding fluid which will induce a longitudinal pressure gradient. If the induced pressure gradient is greater than the buoyancy force, a convective movement in the direction of decreasing pressure is set up in the fluid adjacent to the surface. The buoyancy force in this situation is acting perpendicular to the direction of fluid motion. The problem has been studied theoretically by Stewartson (1958), Gill (1965), Rotem & Claasen (1969), Pera & Gebhart (1972), and Blanc and Gebhart (1974), among others. In all of these papers, boundary layer approximations are applied, and similarity solutions are obtained for wall temperature being a power function of distance from the leading edge.

The corresponding problem of buoyancy induced flow in a saturated porous medium adjacent to an impermeable wall has received relatively little attention. The first analytical paper dealing with this problem appears to be that of McNabb (1965) who studied free convection in a saturated porous medium above a heated circular impermeable surface with wall temperature being a step function with respect to the radius; boundary layer approximations are invoked and approximate solutions are obtained. In the present paper, we shall study free convection in a saturated porous medium above a heated horizontal impermeable surface or below a cooled horizontal impermeable surface where wall temperature is a power function of distance from the leading edge. The boundary layer approximations similar to those employed by Wooding (1963), McNabb (1965), and Cheng & Minkowyc (1975) are invoked, and similarity solutions for the problem are obtained. The problem has important applications to convective flow above the heated bedrock or below the cooled caprock in a liquid-dominated geothermal reservoir.

II. Analysis

Consider the problem of free convection in a saturated porous medium above a heated horizontal impermeable surface or below a cooled surface. The physical situation is shown in Fig. 1 where x and y are Cartesian coordinates in horizontal and vertical directions with positive y axis pointing toward the porous medium. The origin of the coordinate is chosen at the point on the impermeable surface where wall temperature begins to deviate from that of the surrounding fluid. If we assume that (i) the convective fluid and the porous medium are everywhere in local thermodynamic equilibrium, (ii) the temperature of the fluid is everywhere below boiling point, (iii) properties of the fluid and the porous medium are constant, and (iv) the Boussinesq approximation is employed, the governing equations are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad , \quad (1)$$

$$u = -\frac{K}{\mu} \frac{\partial p}{\partial x} \quad , \quad (2)$$

$$v = -\frac{K}{\mu} \left(\frac{\partial p}{\partial y} \pm \rho g \right) \quad , \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad , \quad (4)$$

$$\rho = \rho_{\infty} [1 - \beta (T - T_{\infty})] \quad , \quad (5)$$

where the "+" sign in Eq. (3) refers to the case of a heated plate facing upward (Fig. 1b) while the "-" sign refers to the case of a cooled plate facing downward (Fig. 1a). In Eqs. (1-5), u and v are the velocity components in the horizontal and vertical directions, ρ , μ , and β are the density, viscosity, and the thermal expansion coefficient of the convecting fluid, K is the permeability of the porous medium, $\alpha \equiv k/(\rho C)_f$ is the equivalent thermal diffusivity with k denoting the thermal conductivity of the saturated porous medium and $(\rho C)_f$ the product of density and specific heat of the convecting fluid. T , p , and g are respectively the temperature, pressure, and the gravitational acceleration. The subscript " ∞ " refers to the condition at infinity.

The boundary conditions for the problem are

$$y = 0, \quad T_w = T_\infty \pm Ax^\lambda, \quad v = 0, \quad (6a,b)$$

$$y \rightarrow \infty, \quad T = T_\infty, \quad u = 0, \quad (7a,b)$$

where $A > 0$ and the "+" and "-" signs in Eq. (6a) are for a heated plate facing upward and for a cooled plate facing downward respectively. Eq. (6a) shows that the prescribed wall temperature is a power function of distance from the origin.

The continuity equation is automatically satisfied by introducing the stream function ψ as

$$u = \frac{\partial \psi}{\partial y}, \quad \text{and} \quad v = - \frac{\partial \psi}{\partial x}. \quad (8)$$

Eliminating p from Eqs. (2) and (3) by cross differentiation, the resulting equation in terms of ψ is

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{K\rho_\infty g\beta}{\mu} \frac{\partial T}{\partial x} \quad (9)$$

In terms of ψ , Eq. (4) can be rewritten as

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha} \left(\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} \right) \quad (10)$$

The appropriate boundary conditions for Eqs. (9) and (10) are

$$y = 0, \quad T_w = T_\infty + Ax^\lambda, \quad \frac{\partial \psi}{\partial x} = 0; \quad (11a,b)$$

$$y \rightarrow \infty, \quad T = T_\infty, \quad \frac{\partial \psi}{\partial y} = 0. \quad (12a,b)$$

III. Similarity Solution

From the numerical solutions for free convection in a geothermal reservoir (Cheng, Yeung & Lau, 1975), it is observed that thermal and momentum boundary layers exist along horizontal impermeable surfaces whenever wall temperature differs from that of the surrounding fluid. If boundary layer approximations are invoked, Eqs. (9) and (10) become

$$\frac{\partial^2 \psi}{\partial y^2} = \frac{K\rho_\infty g\beta}{\mu} \frac{\partial T}{\partial x}, \quad (13)$$

and

$$\frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha} \left(\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} \right) \quad (14)$$

To seek similarity solutions to Eqs. (13) and (14) with boundary conditions (11) and (12), we now introduce the following dimensionless variables

$$\eta = \left(\frac{K\rho_{\infty}g\beta A}{\mu\alpha} \right)^{\frac{1}{3}} yx^{\frac{\lambda-2}{3}} = (Ra_x)^{\frac{1}{3}} \frac{y}{x}, \quad (15)$$

$$\psi = \alpha \left[\frac{K\rho_{\infty}g\beta A}{\mu\alpha} \right]^{\frac{1}{3}} x^{\frac{1+\lambda}{3}} f(\eta) = \alpha (Ra_x)^{\frac{1}{3}} f(\eta), \quad (16)$$

$$\theta(\eta) = (T - T_{\infty}) / (T_w - T_{\infty}), \quad (17)$$

where $Ra_x \equiv \rho_{\infty}g\beta K |T_w - T_{\infty}| x / \mu\alpha$ is the modified local Rayleigh number.

In terms of new variables, it can be shown that the velocity components are given by

$$u = \alpha \left(\frac{K\rho_{\infty}g\beta A}{\mu\alpha} \right)^{\frac{2}{3}} x^{\frac{2\lambda-1}{3}} f'(\eta), \quad (18)$$

$$v = -\alpha \left[\frac{K\rho_{\infty}g\beta A}{\mu\alpha} \right]^{\frac{1}{3}} \left[\frac{\lambda-2}{3} \eta f' + \left(\frac{1+\lambda}{3} \right) f \right] x^{\frac{\lambda-2}{3}}. \quad (19)$$

Governing equations (13) and (14) in terms of the new variables are

$$f'' + \lambda\theta + \left(\frac{\lambda-2}{3} \right) \eta\theta' = 0, \quad (20)$$

$$\theta'' - \lambda\theta f' + \left(\frac{1+\lambda}{3} \right) f\theta' = 0, \quad (21)$$

with boundary conditions given by

$$\theta(0) = 1 \quad , \quad f(0) = 0 \quad , \quad (22a,b)$$

$$\theta(\infty) = 0 \quad , \quad f'(\infty) = 0 \quad , \quad (23a,b)$$

where the primes in Eqs. (20-23) denote differentiation with respect to η .

IV. Results and Discussion

Equations (20) and (21) are two coupled non-linear differential equations for θ and f with two-point boundary conditions given by Eqs. (22) and (23). Numerical solutions can be obtained by the Range Kutta method by first converting the boundary-value problem to an initial-value problem and with a systematic guessing of slopes at $\eta = 0$ by the shooting method. Results for f , f' , θ , and θ' vs. η for selected values of λ are presented in Figs. 2-7.

The boundary layer approximations used in the analysis are valid if (i) $\frac{\partial}{\partial y} \gg \frac{\partial}{\partial x}$ and (ii) $v \ll u$. From Eq. (15), it follows that $y/x = O(Ra_x^{-1/3})$. Furthermore, the ratio of Eqs. (19) and (18) gives $v/u = O(Ra_x^{-1/3})$. Thus, the first and the second conditions are satisfied if Ra_x is large. Near the leading edge at $x = 0$, the boundary layer approximations are not expected to be valid. The expressions for thermal and momentum boundary layer thickness can be obtained if the edges of the boundary layers are defined as the points where θ or u/u_w (with u_w denoting the "slip velocity" along the wall) have a value of 0.01. From Figs. 2 and 4 we locate the edges of the boundary layers and denote these values by η_m and η_T . It follows from Eq. (15) that

$$\frac{\delta_m}{x} = \frac{\eta_m}{(Ra_x)^{1/3}} \quad , \quad (24a)$$

$$\frac{\delta_T}{x} = \frac{\eta_T}{(Ra_x)^{1/3}} \quad , \quad (24b)$$

where the values of η_m and η_T for selected values of λ are tabulated in Table 1, which shows that the momentum boundary layer thickness and the thermal boundary layer thickness have about the same order of magnitude.

It is of interest to note that although $u \rightarrow 0$ outside the momentum boundary layer, the value of vertical velocity in general is not zero there. This can be seen from Eq. (19) with (23b) to give

$$v_\infty = -\frac{(1+\lambda)\alpha}{3} \left[\frac{K\rho_\infty g\beta A}{\mu\alpha} \right]^{1/3} f(\infty) x^{\frac{\lambda-2}{3}}, \quad (25)$$

which shows that v_∞ is negative if $\lambda > -1$, positive if $\lambda < -1$, and zero if $\lambda = -1$. Furthermore, the magnitude of v_∞ is increasing with x if $\lambda > 2$, decreasing with x if $\lambda < 2$, and independent of x if $\lambda = 2$. It is worth noting that the value $f(\infty)$ in Eq. (25) is finite as shown in Fig. 2.

To obtain the pressure distribution, we substitute Eqs. (2), (8) and (17) into Eq. (13) and integrate the resulting expression from $x=0$ to x , and from y to $y=\infty$ to give

$$p(x,y) = \bar{p}_\infty g y + p_1(x,y), \quad (26a)$$

with

$$p_1(x,y) \equiv \mu\alpha \left[\frac{\rho_\infty g\beta A}{\mu\alpha} \right]^{2/3} x^{\frac{2(\lambda+1)}{3}} \int_\eta^\infty \theta(\eta) d\eta, \quad (26b)$$

where p_1 is the pressure induced by the density gradient. Along the wall at $y=0$, Eq. (26a) reduces to

$$p(x,0) = p_w(x) = \mu\alpha \left[\frac{\rho_\infty g\beta A}{\mu\alpha} \right]^{2/3} x^{\frac{2(\lambda+1)}{3}} \int_0^\infty \theta(\eta) d\eta, \quad (26c)$$

which shows that p_w is increasing, decreasing, or constant with respect to x depending on whether $\lambda > -1$, $\lambda < -1$, or $\lambda = -1$.

The local surface heat flux can be computed from

$$q = -k \left(\frac{\partial T}{\partial y} \right)_{y=0} \quad (27)$$

With the aid of Eqs. (17) and (15), Eq. (27) can be rewritten as

$$q(x) = kA^{4/3} \left[\frac{K\rho_{\infty}g\beta}{\mu\alpha} \right]^{1/3} x^{\frac{4\lambda-2}{3}} [-\theta'(0)] \quad , \quad (28)$$

which shows that $q(x)$ increases as x is increased if $\lambda > 1/2$; $q(x)$ decreases as x is increased if $\lambda < 1/2$; and $q(x)$ is constant if $\lambda = 1/2$.

We now examine the range of λ for which the problem is physically realistic. Since the wall temperature is different from that of the surrounding fluid at $x > 0$, both u and δ must be increasing or at least constant with respect to x . It follows from Eqs. (18) and (25) that these conditions are satisfied if $1/2 \leq \lambda \leq 2$. Let's consider the variation of boundary layer thickness, vertical velocity at infinity, local surface heat flux, induced pressure and horizontal velocity at the wall with respect to x , as given by Eqs. (18) and (24-28), for the limiting cases of $\lambda = 1/2$ and $\lambda = 2$. The case of $\lambda = 1/2$ corresponds to the constant heat flux case where $u_w = \text{constant}$, $\delta \propto \sqrt{x}$, $v_{\infty} \propto 1/\sqrt{x}$, and $p_w \propto -x$. For the case of $\lambda = 2$, both δ and v_{∞} are independent of x while $q \propto x^2$, $p_w \propto -x^2$, and $u_w \propto x$.

From the definition of the local Nusselt number $Nu_x \equiv hx/k = \frac{qx}{k(T_w - T_{\infty})}$ (where h is the local heat transfer coefficient) and with the aid of Eq. (28), we have

$$\frac{Nu_x}{(Ra_x)^{1/3}} = [-\theta'(0)] \quad , \quad (29)$$

which is presented for selected values of λ in Table 1 and plotted in Fig. 6.

The overall surface heat transfer rate for a rectangular surface with a length L and a width S can be computed from

$$Q = S \int_0^L q(x) dx \quad , \quad (30)$$

which can be integrated explicitly after $q(x)$, given by Eq. (28), has been substituted in Eq. (30) to give

$$Q = \left(\frac{3}{4\lambda+1}\right)[- \theta'(0)] S k A^{4/3} \left[\frac{\rho_{\infty} g \beta K}{\mu \alpha}\right]^{1/3} L^{\frac{4\lambda+1}{3}} . \quad (31)$$

The average Nusselt number is defined by $\overline{Nu} = \overline{h}L/k$ where the average heat transfer coefficient \overline{h} depends on the choice of the temperature difference between the wall and the temperature of the fluid away from the wall. If the temperature difference is based on the mean temperature difference defined by

$$\overline{(T_w - T_{\infty})} = \frac{1}{L} \int_0^L (T_w - T_{\infty}) dx = \frac{AL^{\lambda}}{1+\lambda} = \frac{(T_w - T_{\infty})L}{1+\lambda} , \quad (32)$$

and

$$Q = \overline{h}(\overline{T_w - T_{\infty}})SL , \quad (33)$$

then, the average Nusselt number is given by

$$\frac{\overline{Nu}}{(Ra)^{1/3}} = \frac{3(1+\lambda)^{4/3}}{(1+4\lambda)} [- \theta'(0)] , \quad (34)$$

where $Ra \equiv |\overline{T_w - T_{\infty}}| \rho_{\infty} g \beta K L / \mu \alpha$. Eq. (34) for different values of λ is presented in Table 2 and in Fig. 7.

It will be of interest to compare free convection in a saturated porous medium adjacent to a vertical flat plate with that of a horizontal plate. The corresponding expressions for thermal boundary layer thickness and the average Nusselt number for free convection about a vertical flat plate embedded in a saturated porous medium are (Cheng & Minkowycz, 1975)

$$\frac{\delta_T}{x} = \frac{\eta_T}{(Ra_x)^{1/2}} , \quad (35)$$

and

$$\frac{\overline{Nu}}{(Ra)^{1/2}} = \frac{2(1+\lambda)^{3/2}}{(1+3\lambda)} [-\theta'(0)] \quad (36)$$

A comparison of Eqs. (24a) and (34) to Eqs. (35) and (36) shows that $(\delta_T)_v < (\delta_T)_h$ and $(\overline{Nu})_v > (\overline{Nu})_h$ where the subscripts v and h denote the vertical flat plate and a horizontal flat plate respectively. To gain some feeling of the order of magnitude of various physical quantities in a geothermal application, consider an upward facing heated horizontal impermeable surface, 1 km by 1 km, with a mean wall temperature of 573°K embedded in an aquifer at 288°K. For numerical computations the following physical properties are used: $\beta = 2.8 \times 10^{-4}/^\circ\text{K}$, $\rho_\infty = 0.92 \times 10^6 \text{ g/m}^3$, $C = 4.2 \times 10^3 \text{ Joule/kg-}^\circ\text{K}$, and $k = 2.4 \text{ Watt/m-}^\circ\text{K}$. The value of μ is a strong function of temperature varying from $.54 \times 10^{-3} \text{ Newton-sec/m}^2$ at 288°K to $0.042 \times 10^{-3} \text{ Newton-sec/m}^2$ at 573°K, whereas the value of K depends on the locality ranging from 10^{-14} m^2 at Wairakei, New Zealand, to 10^{-10} m^2 at Hawaii. If the value of $\mu = 0.54 \times 10^{-3} \text{ Newton-sec/m}^2$ and $K = 10^{-12} \text{ m}^2$ are used, it is found that the boundary layer thickness increases from zero at the origin of the horizontal heating surface to approximately 350m at 1 km, and the total heat transfer rate is approximately 10 MW. For a vertical heating surface of the same size, the boundary layer thickness increases from zero at the origin to 170m at 1 km with a total heat transfer rate of 30 MW. If the values of $\mu = 0.042 \times 10^{-3} \text{ Newton-sec/m}^2$ and $K = 10^{-10} \text{ m}^2$ are used, the boundary layer thickness will be considerably thinner with an associated increase in heat transfer rate.

V. Concluding Remarks

The foregoing analysis is based on the boundary layer approximations which are applicable for large Rayleigh numbers. The analytical expressions for total surface heat transfer can be used for an approximate estimate of

energy transfer rate between a horizontal surface to the surrounding saturated porous medium when the temperature of the impermeable surface is different from that of the surrounding fluid. The first author (P. Cheng) is currently extending the analysis to an axisymmetric flow in a porous medium heated or cooled by a circular impermeable surface. Results of the analysis will be presented at a later time.

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Table 1 Values of $[-\theta'(0)]$, η_T , and η_m for Selected Values of λ

λ	$[-\theta'(0)]$	η_T	η_m
0.5	0.8164	5.0	6.4
1.0	1.099	4.5	5.4
1.5	1.351	4.0	4.4
2.0	1.571	3.7	3.8

Table 2 Values of $Nu/(Ra)^{1/3}$

λ	$Nu/(Ra)^{1/3}$
0.5	1.402
1.0	1.662
1.5	1.965
2.0	2.266

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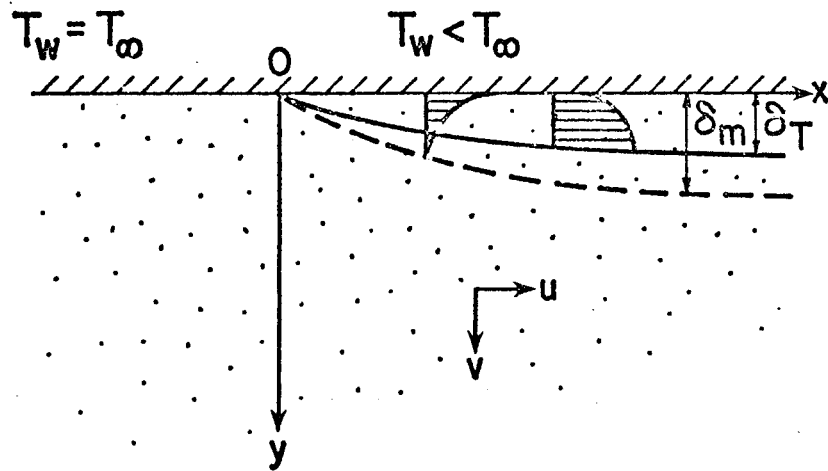


Fig. 1a A Cooled Surface Facing Downward

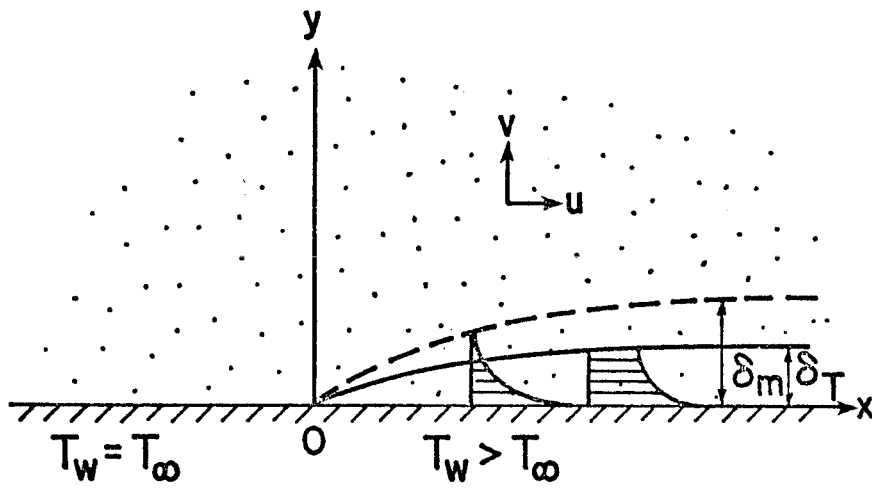


Fig. 1b A Heated Surface Facing Upward

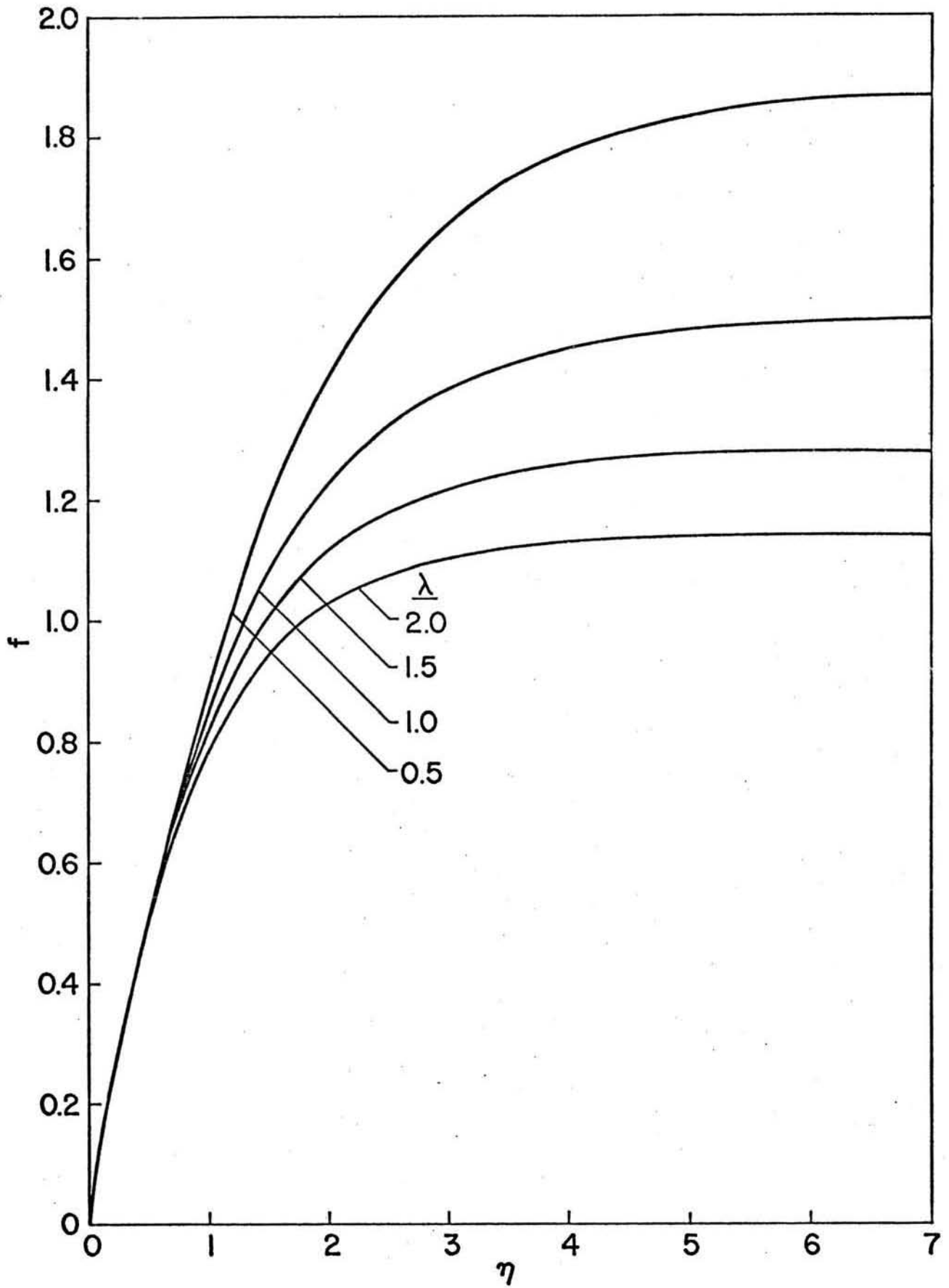


Fig. 2 Value of f Versus η

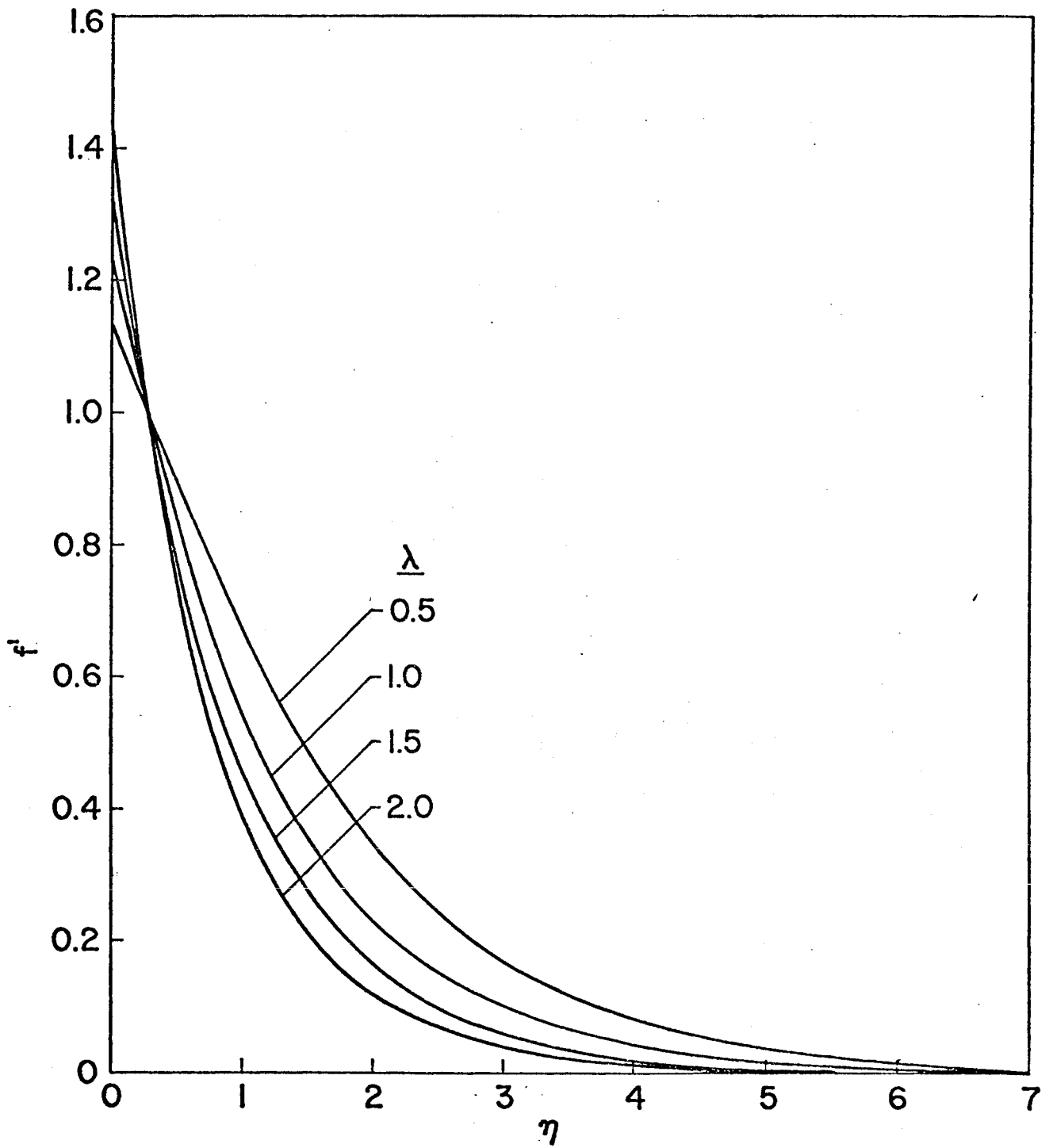


Fig. 3 Dimensionless Velocity Distribution
Versus η for Selected Values of λ

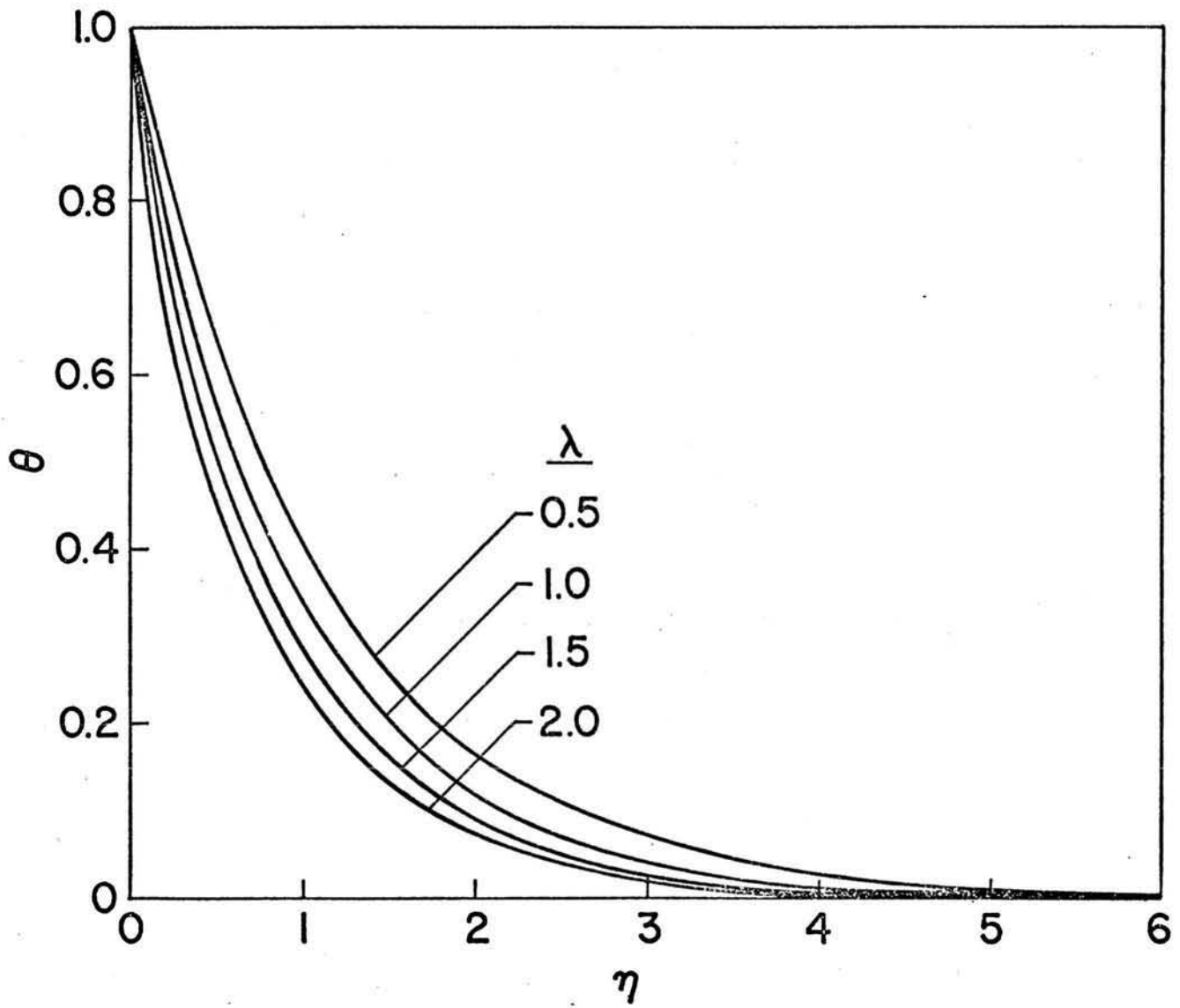


Fig. 4 Dimensionless Temperature Versus η for Selected Values of λ

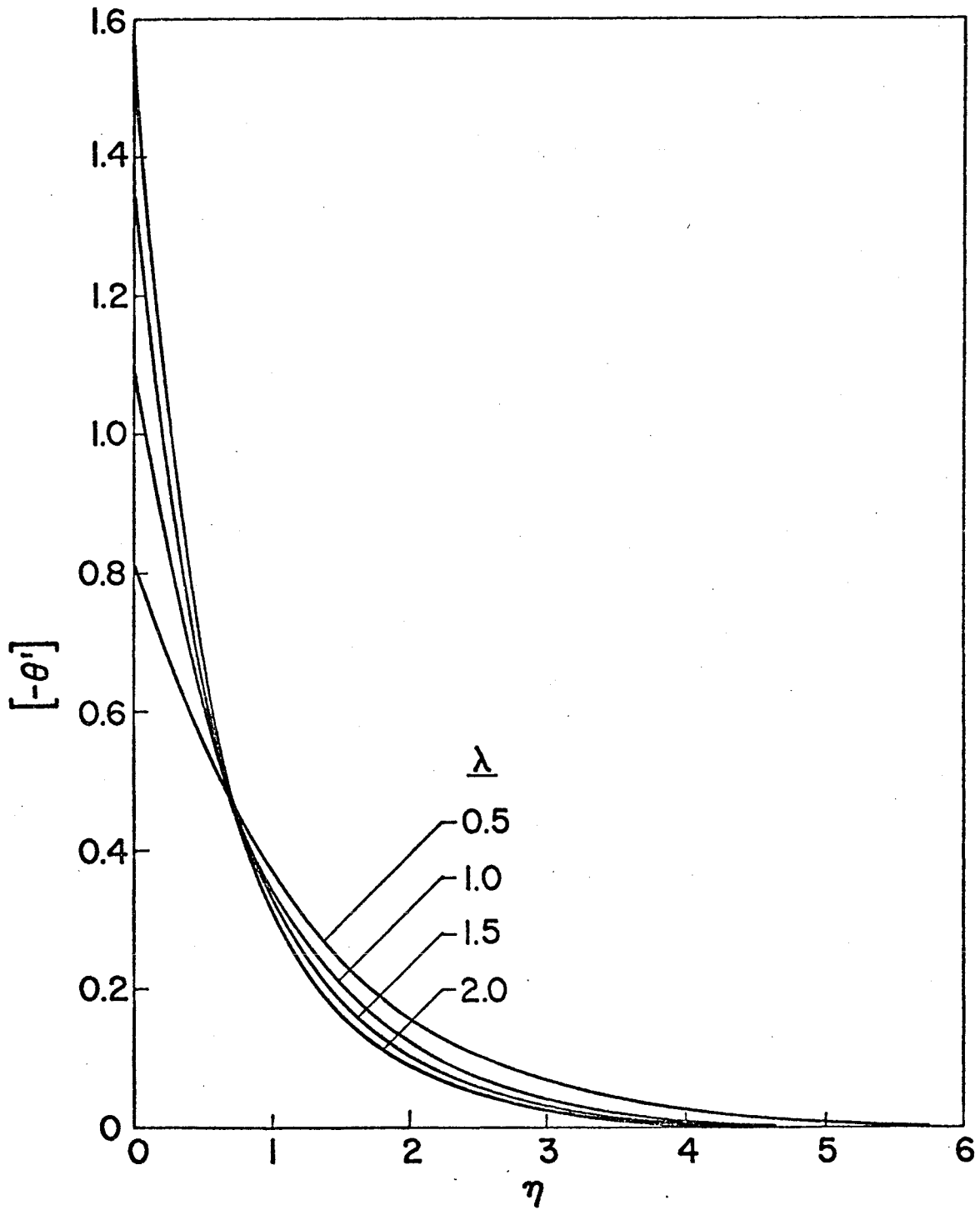


Fig. 5 Value of $[-\theta']$ Versus η

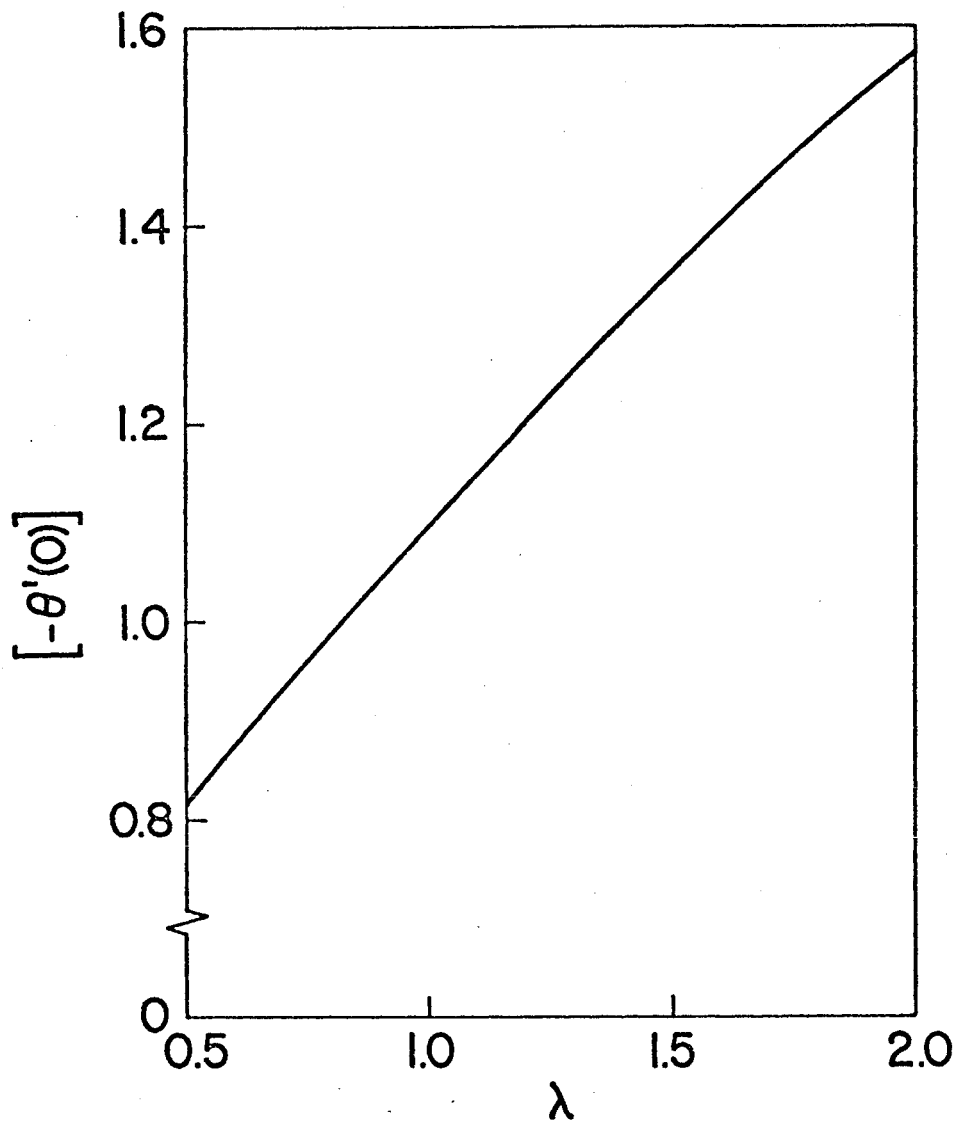


Fig. 6 Values of $Nu_x / (Ra_x)^{1/3}$ or $[-\theta(0)]$ Versus λ

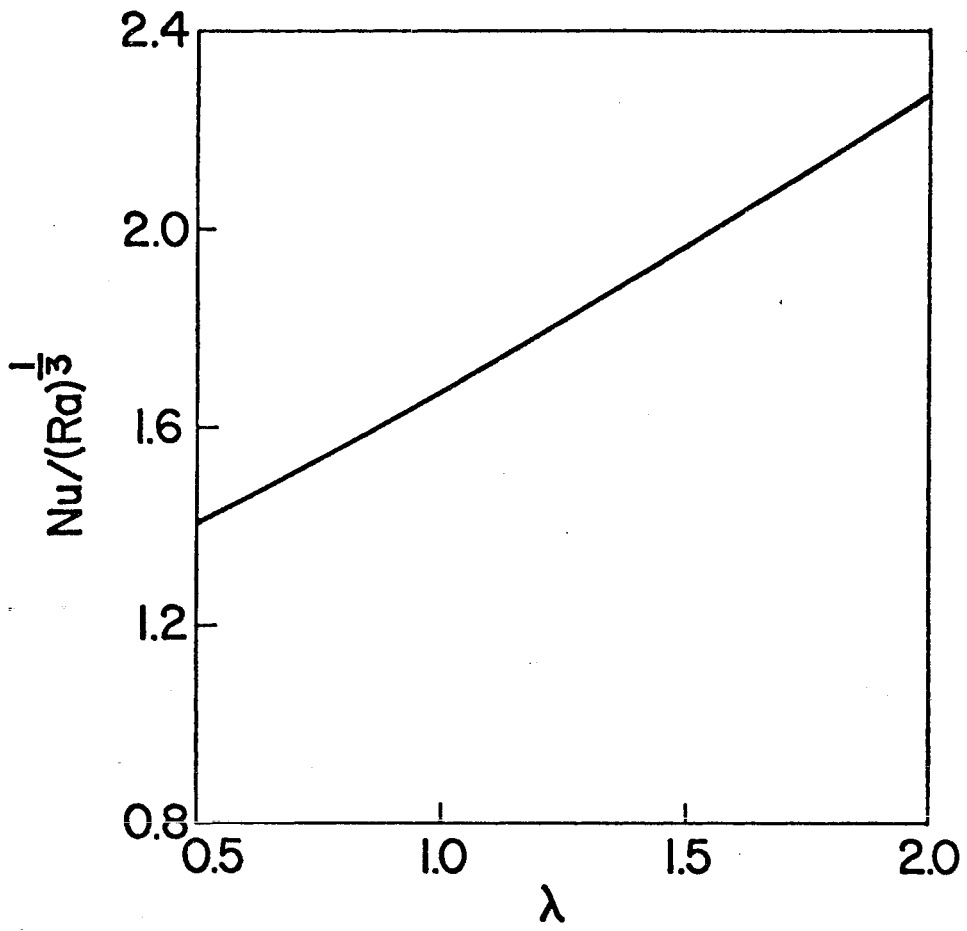


Fig. 7 Values of $\overline{Nu}/(Ra)^{1/3}$ Versus λ